Synopsis for November 16 2025

Geometry of circles

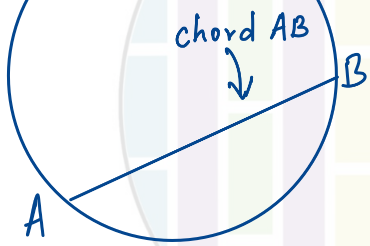
Basic terminology

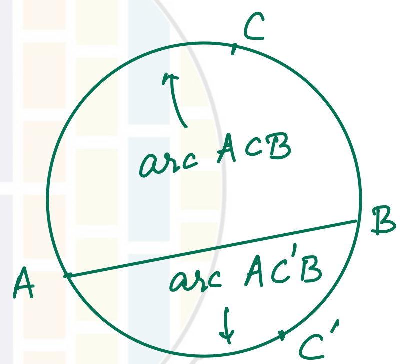
In this session we started working on the geometry of circles. Students who are new to this topic need to know 3 ideas/concepts to begin working with circles. These are (1) the definition of a circle, (2) chords and arcs and (3) the meaning of *subtended angles*.

We discussed the definition of a circle as *the locus of points that lie at a fixed distance from a fixed point* and understood that the fixed point is the *centre* and the fixed distance is the *radius* of the circle.

If A and B are any two points on a given circle, then the line segment joining these points is called a *chord*. It is denoted as 'chord AB'.

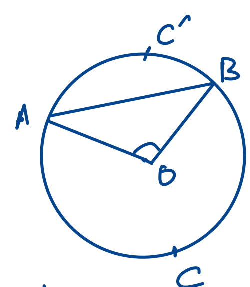
The points A and B divide the circumference into two *arcs*. To label each arc, we can choose a third point, say C (respectively C') on each arc and label the arcs as 'arc ACB' and 'arc $AC'B$'.





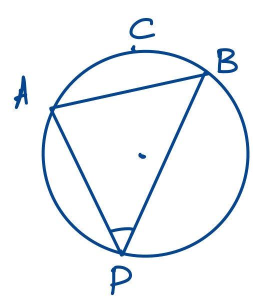
Any chord or arc can *subtend* angles at the center or at any point on the circumference of the given circle as shown in the figures below:

1. Angle subtended by chord AB at center $O = \angle AOB$. This is also the angle subtended at the center by arc $AC'B$



2. The angle subtended by arc ACB at the center is $360° - \angle AOB$.

3. For any point P on the circle, $\angle APB$ is the angle subtended by chord AB (or arc ACB) at P.



Results

We discussed and proved the following results:

Result 1 The perpendicular bisector of any chord of a circle passes through the center of the circle.

Result 2 Given any three points $A, B.C$, there exists a unique circle passing through A, B and C.

Remarks: Note that the above results tells us:

(a) If two circles have at least 3 points in common, then the circles must coincide (that is, they must be one and the same circle).

(b) Two distinct circles cannot intersect in more than two points.

(c) Two circles cannot have a common arc unless they coincide.

Result 3 Equal chords of a circle are equidistant from the center.

Result 4 (State and prove the converse of Result 3)

Result 5: For any arc of a circle, the angle subtended by the arc at the center is twice the angle subtended at any point on the remaining part of the circle.

Corollary 1 Angles in the same segment of a circle are equal

Corollary 2 The angle subtended by a semicircle is 90°.

Result 6: If a straight line segment joining two points on a circle subtends equal angles on the same side of it, then the four points are concyclic.

Math with paper folding

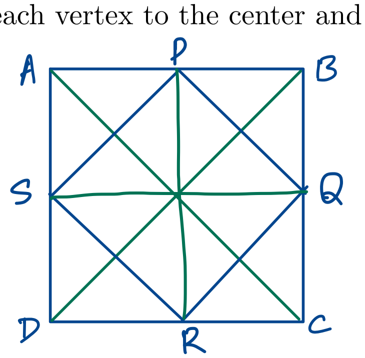
We proved that the infinite series $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots$ equals 1 using paper folding. Here's the outline of the method:

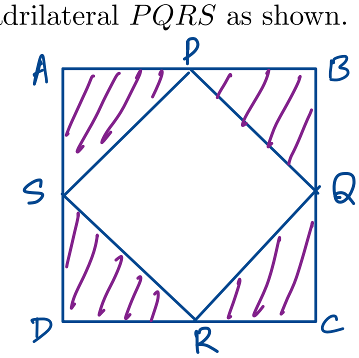
1. Take a square piece of paper, and we will assume its side length is 1 unit. Label this as square $ABCD$.

2. Fold the diagonals and the half lines (in both directions).

3. Now fold each vertex to the center and form the quadrilateral $PQRS$ as shown.

Area($\square ABCD$) $= 1.$





Area($\square PQRS$) $= \frac{1}{2}$ = area of the shaded region.

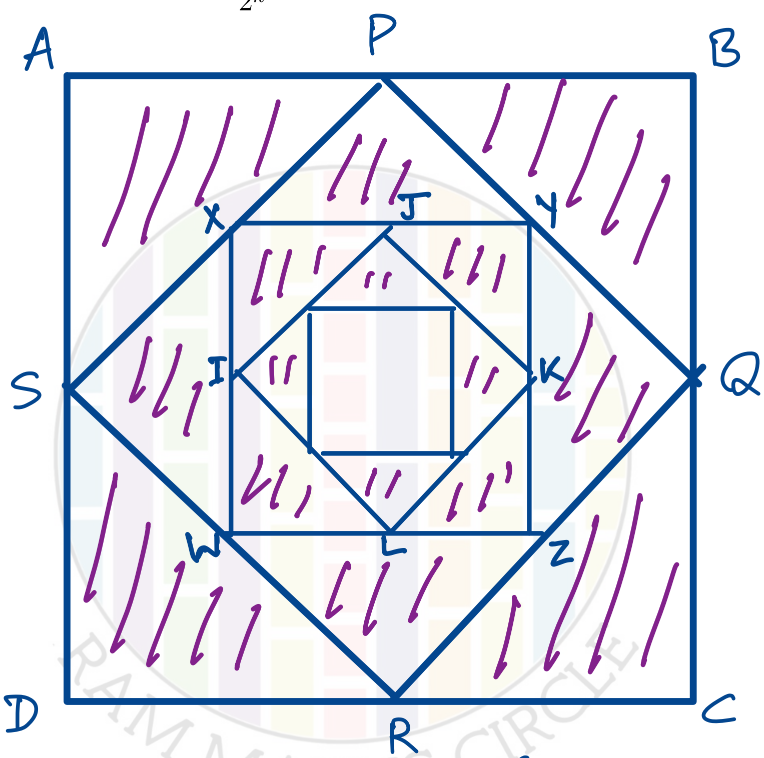
4. Show that $PQRS$ is a square whose area is half that of the original square. In addition, area of $PQRS$ is also equal to the area of the flaps that fold on to it, i.e. sums of areas of triangles APS, PBQ, QCR and RDS.

5. Repeat this process with $PQRS$ and the starting square, to obtain a square whose area is $\frac{1}{4}^{th}$ of the original square.

6. Continue repeating the process to get nested squares within earlier squares as shown.

7. At each step, the area of the innermost square is equal to the sum of the areas of the triangular flaps that fold on to it, as well as it is equal to half the area of the earlier square in the sequence.

8. If we consider the formation of square $PQRS$ as our first step, then at the k^{th} step, the area of the innermost square formed is $\frac{1}{2^k}$.



Area of the shaded region (counting inwards from the outermost triangular flaps)

$= \text{Area}(\square PQRS) + \text{Area}(\square XYZW) + \text{Area}(\square IJKL) + \cdots$

$= \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots$

Continuing this way, the shaded region encompasses the entire square whose area is 1 !