RAM Maths Circle

July 20, 2025

Nagpur

Introduction

In this session, the students were introduced to **Graph Theory** with intuitive problems. The students were quick to grasp the elements of problems, relating them easily with the concepts of Graph Theory. The students actively engaged in bringing up all possible solutions to the problem. The session concluded with a colouring activity to arrive at the proof for the solution.

Activity 1: Drawing Without Lifting the Pen

Observation: A figure can be drawn without lifting the pen if each edge is traversed exactly once.
Graph Interpretation: Vertices represent points; edges represent lines between them. Example graph:

$$V = \{1,2,3,4\}, \quad E = \{(1,2),(2,3),(3,4),(4,1)\}$$

Problem 1: The Room Layout Problem

Problem: Given a house layout, is it possible to visit each room exactly once?

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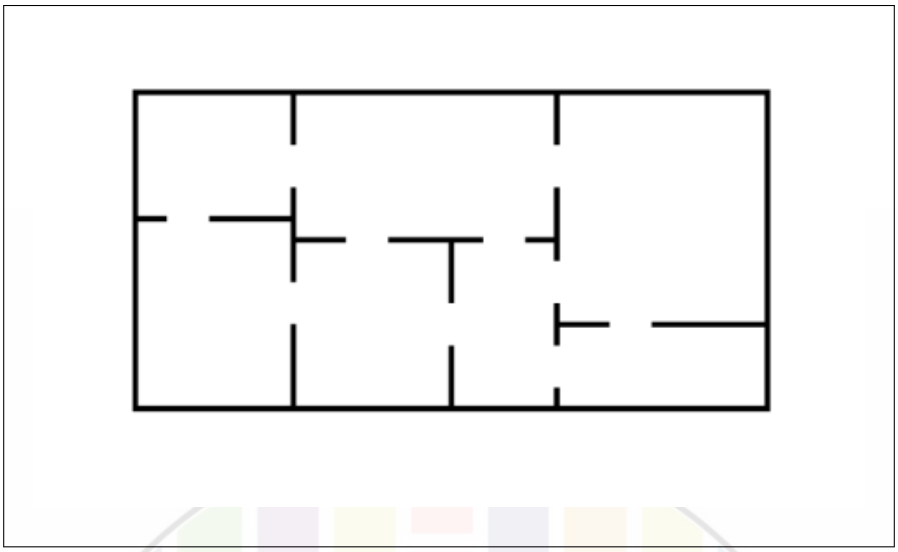

Students observed that the solution formed a cyclic series, allowing them to find different variations of the solution, for each of the possible starting points.

Graph Representation

$$\text{Rooms} \to \text{Vertices}, \quad \text{Doors} \to \text{Edges}$$

After listing all vertices and edges, students successfully drew the corresponding graph. They confirmed that it is possible to traverse every edge exactly once — linking back to the initial "pen-lift" puzzle.

Problem 2: The Art Gallery Problem

Problem: Given a polygon-shaped gallery and 360° cameras placed only on vertices, find the minimum cameras required to cover the entire gallery.

Key Observations

- Place cameras on vertices that can "see" the maximum other vertices.
- Triangulate the polygon by drawing non-intersecting diagonals.
- Number of cameras that will always be sufficient: $\lfloor n/3 \rfloor$ for an n-sided polygon.

Example: A 6-sided polygon will always be covered with 2 cameras.

Coloring Interpretation

Vertices can be grouped into 3 disjoint color sets (3-coloring of the triangulated graph). Each color class corresponds to one camera placement.

Task for Students

1. What if we wanted to guard the **exterior** of the polygon instead of the interior?
2. Can we still use vertex cameras?
3. Do we need more cameras or fewer?
4. In reference to the House Layout Problem, is it possible to walk through **every doorway (edges) exactly once** without repeating any? If so, in which room(s) must the tour begin and end? Explain your reasoning.
5. Why, if such a path exists, must it pass through at least one room (vertex) **more than once** when visiting all doors or it can be avoided?

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