RAM Math Circle - Chennai

Synopsis for August 3 2025

1 Recall

Till now we have seen 5 strategies for counting:

- The addition principle
- The multiplication principle
- Number of arrangements of n objects taken r at a time, with repetition allowed.
- Permutations or arrangements
- Combinations or selections

Today we will practice these with some variations and also see another interesting strategy.

1. How many ways are there to place 2 identical rooks in a common row or column of an 8×8 chessboard?
2. How many ways are there to place 2 identical queens on an 8×8 chessboard so that the queens are not in a common row, column or diagonal?
3. How many different letter sequences can be obtained by rearranging the letters of the given words:
   (a) PARENT
   (b) TRUST
   (c) CARAVAN
   (d) MATHEMATICS
4. There are 20 towns in a certain country and every pair of them is to be connected by an air route. How many air routes will there be?
5. How many diagonals are there in a convex $n-$gon?

2 Circular permutations

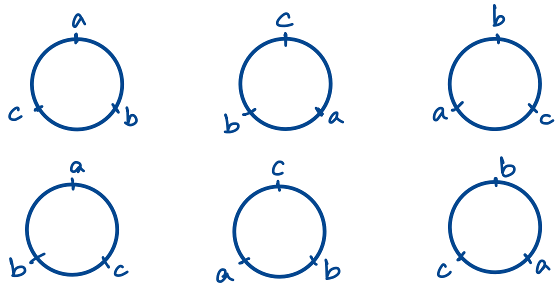
A *circular permutation* means an ordered arrangement of the given elements around a circle.

There are $3! = 6$ different permutations of 3 people in a row. These are

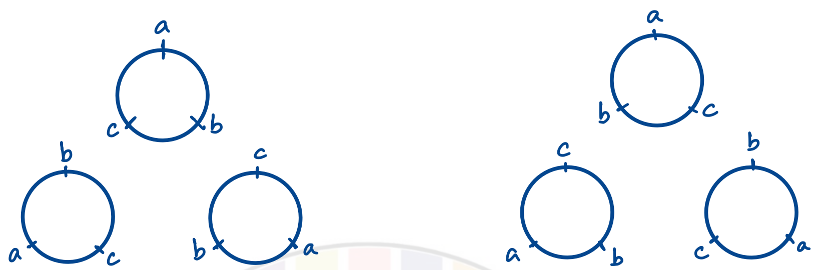
$$a, b, c \qquad c, a, b \qquad b, c, a \qquad a, c, b \qquad c, b, a \qquad b, a, c$$

But suppose that the people are seated around a circular table and suppose that the seats are *not* numbered. Then in how many different ways can they be seated? This is same as asking "how many different circular permutations of a, b, c are there?"

Note that the arrangements now look like this:



Some of these arrangements can be obtained from the others by a rotation. We can group these arrangements as follows:



So the truly distinct permutations, that is those that are not related by a rotation, are:

Notice that we can get these in the following way: fix a position for a, then the remaining two people can be arranged in the two seats in $2!$ ways.

In general, the number of distinct circular permutations of n different objects is $(n-1)!$.

2.1 Practice set

1. Let $n \geq 3$ be an integer. Prove that the number of different circular necklaces that can be made from n different beads is $\frac{1}{2}(n-1)!$.

2. There are 12 members in a committee who sit around a table. There is one place specially designated for the chairman. Besides the chairman there are 3 people who constitute a subcommittee. Find the number of seating arrangements if

   (a) the subcommittee members sit together as a block.

   (b) no two of the members of the subcommittee sit next to each other.

3. How many different necklaces can be made using 7 beads of which 5 are identical red beads and 2 are identical blue beads?