
IIIT Delhi - RAM Maths Circle

Session 8 and 9

(Organized by the Department of Mathematics, IIIT Delhi)

IIIT Delhi

September 20th and 27th, 2026

Turning Geometry into Algebra

We humans love patterns. But what is the language to express them?

Instruction

The purpose of this session is not to give you answers, but to leave you with better questions—questions that mathematics will not let you forget.

Discussion Rules

- The problem sheet will be distributed at the very beginning of the session.
- I will solve the easiest question of the sheet.
- The remaining questions are for discussion. You will work on them, discuss your approaches, and develop the solutions yourselves.
- Different approaches are welcome. A wrong approach, if discussed carefully, can be as valuable as a correct one.
- Do not hesitate to ask questions, challenge an argument, or propose an alternative solution.

Question 1: The HIV Outer Shell

The picture below shows the outer shell of the cross section of the human immunodeficiency virus (HIV).



Find the number of rotational symmetries, the number of reflective symmetries, and the total number of symmetries of the outer shell. Draw a composition table for all the symmetries.

Question 2: The Clock of Seven Numbers

Imagine a clock with only seven positions: 0, 1, 2, 3, 4, 5, 6. After 6, the clock comes back to 0. We can think of this as what happens to the remainders when numbers are divided by 7. Let

$$S = \{0, 1, 2, 3, 4, 5, 6\}.$$

Now let us create a new rule for adding the numbers on this clock. Take two numbers a and b from S .

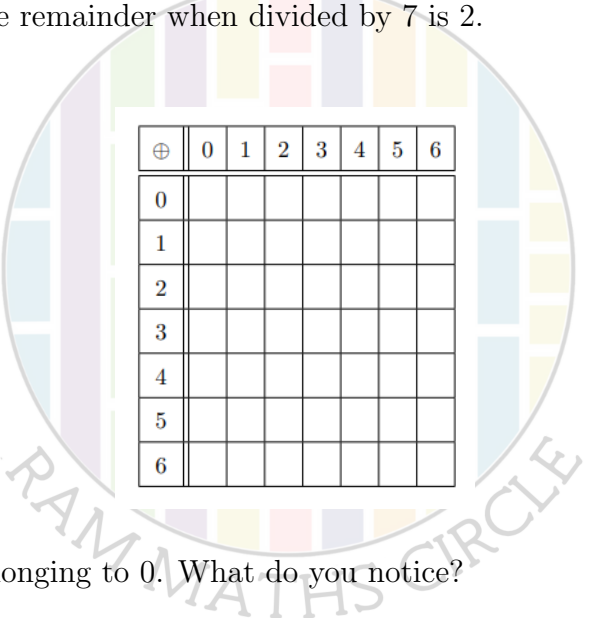
Add them in the usual way and then keep only the remainder after dividing the answer by 7. Call this new answer $a \oplus b$.

For example,

$$5 \oplus 4 = 2$$

because $5 + 4 = 9$ and the remainder when divided by 7 is 2.

Complete the table:



$\oplus$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

1. Look at the row belonging to 0. What do you notice?
2. Is there a number which leaves every other number unchanged when added using this rule? Find it.
3. Pick any number from S . Can you find another number which, when added to it using this rule, brings you back to 0?
4. Does it matter which number comes first? For example, compare $2 \oplus 5$ and $5 \oplus 2$. Try a few more pairs and describe what you notice.
5. Choose three numbers and compare

$$(a \oplus b) \oplus c \quad \text{with} \quad a \oplus (b \oplus c).$$

Try several choices. What do you think always happens?

6. Start at 0 and keep adding 1 using this new rule:

$$0, \quad 0 \oplus 1, \quad (0 \oplus 1) \oplus 1, \quad \dots$$

Write down the first ten numbers. What pattern do you see?

7. Suppose two numbers leave the same remainder when divided by 7. If we add the same number to both of them, will their remainders still be the same?

You have discovered a new way of adding on a clock with seven numbers.

Question 3: The Never-Ending H Strip

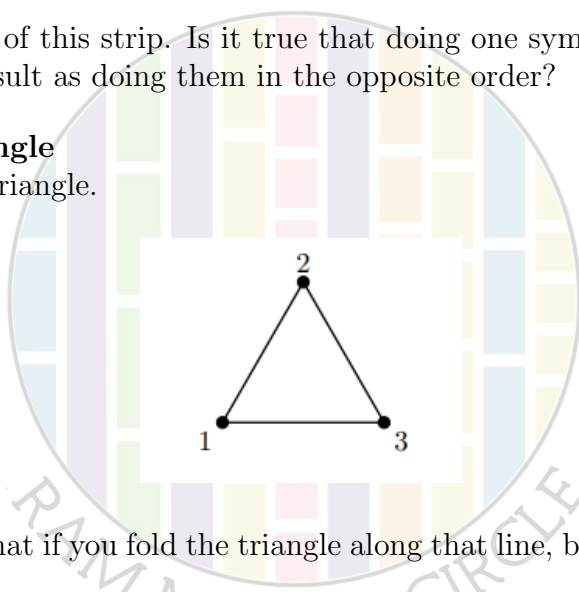
Consider an infinitely long strip of equally spaced H's:

$$\dots H H H H H \dots$$

Describe the symmetries of this strip. Is it true that doing one symmetry followed by another always gives the same result as doing them in the opposite order?

Question 4: The Triangle

Consider an equilateral triangle.

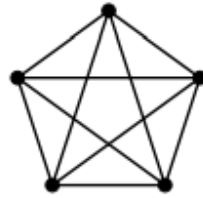


A flip line is a line such that if you fold the triangle along that line, both halves match perfectly.

- How many different flip lines does an equilateral triangle have? Draw them all on the triangle below.
- Now you remove the triangle from its original position, and you want to re-insert it back into the original position. The triangle must fit perfectly.
In how many possible ways can you re-insert the triangle? Name them all.
- Make a table as we did for the square showing the result of doing one way followed by another.
- Find two ways that are opposites of each other— doing one and then the other brings you back to the start. How many such opposite pairs can you find?
- Is doing way A then way B always the same as doing way B then way A? Find an example where it is not the same.
- Can you get every way by using only turns? What about using only flips? Explain.

Question 5: Symmetry of a Star

Here is a 5-pointed star.



If you turn the star around its centre, after some small turns it looks exactly the same again.

- After how many small turns does the star look exactly the same?
- Name each turn-symmetry. For example, let e be “do nothing”, r be “turn once”, r^2 be “turn twice”, and so on.
- How many different turn-symmetries does it have (including “do nothing”)?
- If you turn it 12 times, where does it end up?
- Make a composition table for the star. Fill in what happens when you do one turn-symmetry followed by another.
- What is

$$r \circ r \circ r \circ r \circ r \circ r?$$

What is

$$r^3 \circ r^4?$$

Here $r \circ r$ means we do r followed by r again and re-insert.

- Compare the table you constructed in Question 1 with the table you constructed in this question. List down all the common patterns that you see.

Question 6

Look at these letters:

F G J K L P Q R

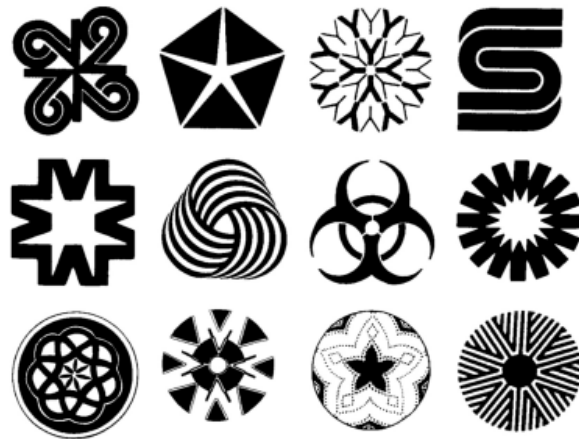
Now look at all the other letters:

A B C D E H I M N O S T U V W X Y Z

In terms of symmetry, what is special about the first group of letters that the second group does not have?

Question 7: Symmetry of a Design

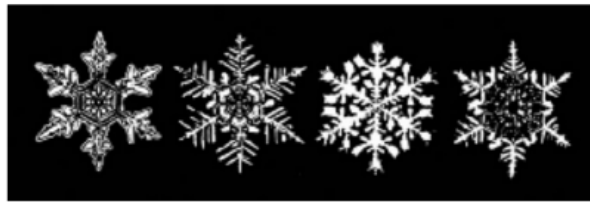
The picture below shows a design.



Find all the symmetries of this design. Then draw a composition table for all the symmetries.

Question 8: Symmetry of Snowflakes

The figure below shows snowflakes. For each snowflake in the figure, find all its symmetries and locate all the axes of reflective symmetry.



Question 9

Let S be the set of all symmetries of a square.

Find three symmetries A , B , and C of the square such that doing A followed by B gives the same result as doing B followed by C , but A is not the same as C .

Question 10: Symmetry of Quadrilaterals

- Describe all the symmetries of a parallelogram that is neither a rectangle nor a rhombus.
- Describe all the symmetries of a rhombus that is not a rectangle.

Question 11

Two symmetries of a square A and B are said to match if doing A followed by B gives the same result as doing B followed by A .

- Find all symmetries that match with every other symmetry.
- Pick the symmetry “turn 90° ”. Find all symmetries that match with it.
- Pick the symmetry “flip left-right”. Find all symmetries that match with it.

- (d) Pick the symmetry “flip along a diagonal”. Find all symmetries that match with it.
- (e) Do you notice any pattern? Which symmetry matches with the most others? Which matches with the fewest?

Question 12

Consider all the symmetries of a square.

Some symmetries have a special property: when you do them with any other symmetry, the order does not matter.

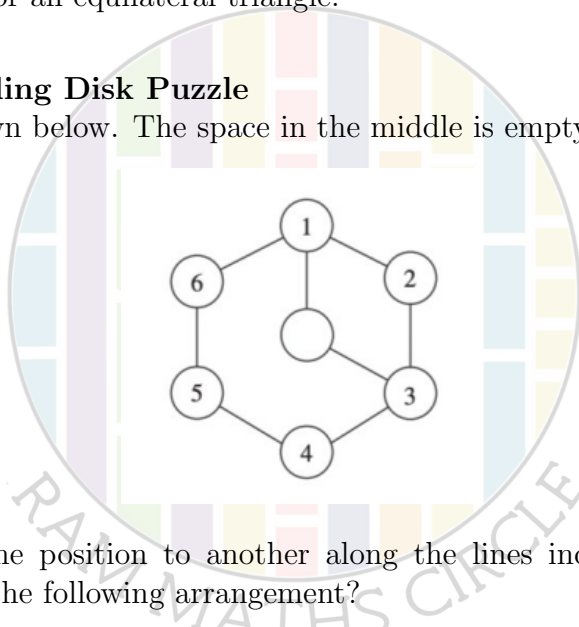
In other words, a symmetry A has this property if, for every symmetry X ,

$$A \text{ followed by } X = X \text{ followed by } A.$$

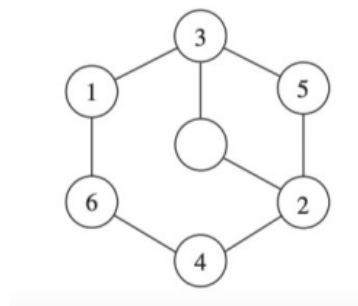
- (a) Find all symmetries of the square that have this property.
- (b) Now do the same for an equilateral triangle.

Question 13: The Sliding Disk Puzzle

Consider the puzzle shown below. The space in the middle is empty.



By sliding disks from one position to another along the lines indicated, without lifting or jumping, can we obtain the following arrangement?



We love the symmetries and patterns around us so much that we cannot take a moment of thought before appreciation.

How do we express it in a beautiful language to make everyone understand your feeling and appreciations equally as you do?