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## IIIT Delhi - RAM Maths Circle

### Session 4

(Organized by the Department of Mathematics, IIIT Delhi)

IIIT Delhi

August 23<sup>rd</sup>, 2026

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## Challenge Problems

**Problem 1.** There are 18 teams in a tournament. The tournament is played over 8 rounds, and no pair of teams plays each other more than once. Prove that there are three teams such that no two of them have played each other.

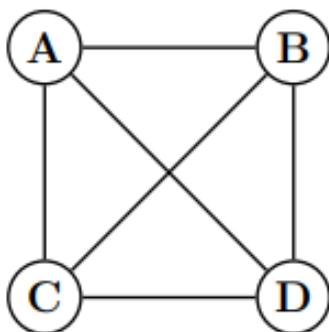
**Problem 2.** Let  $G$  be a graph with  $n$  vertices and  $n + 1$  edges. Prove that there exists at least one vertex whose degree is no less than 3.

**Problem 3.** There are 123 people attending an academic symposium, and each person has discussed with at least 5 other participants. Prove that there is at least one person who has discussed with more than 5 participants.

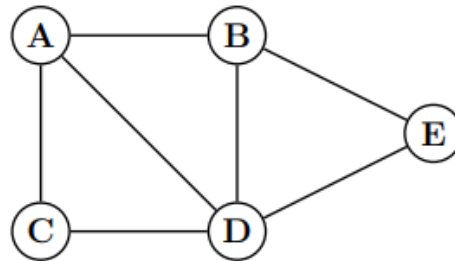
**Problem 4.** Imagine that 6 people are in a room. For every pair of people, exactly one of the following is true: they are friends or they are strangers. Prove that there must be either three mutual friends or three mutual strangers.

Can you find the smallest number of people for which the same conclusion is always true?

**Problem 5.** A designer wants to draw the figure below without lifting the pencil from the paper and without drawing any line more than once. Represent the figure as a graph and determine whether this is possible. If it is possible, give a route that the designer can follow. If it is not possible, explain why.

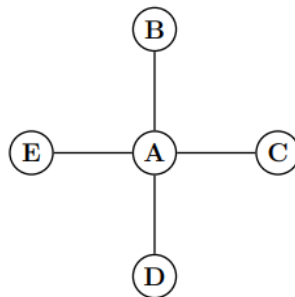


**Problem 6.** A postman has to deliver letters along every street in a small town. He wants to start from the post office, travel along every street at least once, and return to the post office. The post office is located at vertex  $A$ . The streets and their connections are represented by the graph below.



- (a) Can the postman travel along every street exactly once and return to the post office? Explain your answer.
- (b) If this is not possible, what is the minimum number of streets that must be travelled more than once?
- (c) Find a route that starts and ends at  $A$ , covers every street, and uses the minimum possible number of repeated streets.

**Problem 7.** A student wants to draw the figure below in one continuous stroke without lifting the pencil and without drawing any line twice. Unfortunately, the figure is not quite “one-stroke friendly.”



- (a) Explain why the figure cannot be drawn in one stroke.
- (b) You are allowed to add exactly one new line connecting two of the existing vertices. Where could you add this line so that the whole figure can now be drawn in one stroke?
- (c) After adding your new line, find a route that uses every line exactly once.

*Can you find more than one place where the new line could be added?*