

# RAM Maths Circle

November 2, 2025

Nagpur

## Introduction

Modular arithmetic gives us a powerful way to deal with integers by focusing on remainders, enabling us to solve problems more easily than working with large numbers directly. It models naturally “cyclic” or “wrap-around” phenomena (like hours on a clock, days of the week). A useful way to visualise this is a clock: for example with modulus 12, after 12 you wrap back around to 0.

## Problem 1

Try listing **all the numbers from 1–30** which leave remainder 2 when divided by 6. Then try finding some patterns in those numbers.

## Problem 2

Two pairs of congruent numbers for (mod 7):

Find two distinct pairs  $(a, b)$  such that  $a \equiv b \pmod{7}$ .

## Exploration

Suppose

$$a \equiv b \pmod{n} \quad \text{and} \quad c \equiv d \pmod{n}.$$

Thus there exist integers  $k$  and  $\ell$  such that

$$a - b = kn \quad \text{and} \quad c - d = \ell n.$$

Students were wondering how to do it ,one of the students began with this , to see whether it might help them in getting the proof

$$(a - b) - (c - d) = kn - \ell n = xn$$

for some integer  $x$ . They began with this framework to arrive at the conclusion

$$a + c \equiv b + d \pmod{n}$$

if it holds true

### Problem 3

**Prove that:** If

$$a \equiv b \pmod{n} \quad \text{and} \quad c \equiv d \pmod{n},$$

then

$$ac \equiv bd \pmod{n}.$$

### Problem 4

Find the remainder when  $3^{50}$  is divided by 7.

