

Krea - RAM Maths Circle - Session 5

Hurudaya Narasimhan

6/9/2026

1 Overview

In the final session of voting theory, we looked at Arrow's Impossibility Theorem and the proof for it.

2 Election Implementation

- An election was conducted, where each student was asked to rank the method they would prefer for the elections from the 4 types of elections explained last time: Plurality (P), Instant Runoff (I), Borda Count (B) and Condorcet (C).
- Results:
 1. $P > B > C > I$ got 2 votes
 2. $C > B > P > I$ got 3 votes
 3. $C > B > I > P$ got 3 votes
 4. $C > P > B > I$ got 1 vote
 5. $I > P > B > C$ got 4 votes
 6. $P > C > I > B$ got 1 vote
 7. $B > I > C > P$ got 1 vote
 8. $I > C > B > P$ got 1 vote
 9. $B > I > P > C$ got 1 vote
 10. $P > C > B > I$ got 3 votes
 11. $C > P > I > B$ got 2 votes
- Observation: Condorcet method won when the votes were counted through Plurality, Instant Runoff and Borda Count.
- But when it was counted through Condorcet, Condorcet didn't win in the election.

3 Arrow's Impossibility Theorem

With 3 or more candidates and unrestricted voter preferences, not a single voting system at once is

- Pareto
- IIA
- Non-dictatorship while producing a social ranking

4 Proof

If there are V people who can vote. Let $N \subseteq V$ This is the number of people who vote.

1. Decisive, $D \subseteq N$ is when $X > Y$ when all voters in D have $X > Y$. The social ranking will be $X >_s Y$. Set of Decisive groups have to be non zero.
2. Strength of decisive groups. Decisive group is weak if $|D| < |N|$. Decisive group is strong if $|D| = |N|$.
3. Smallest decisive group has one voter. For contradiction lets assume that smallest $|D| \geq 2$. If D splits into D_1 and D_2 where in D_1 it is $A > B$. And in D_2 it is $B > A$. this will mean that D_1 and D_2 both will have 1. Therefore smallest decisive group is 1, $|D| \geq 1$.