
IIIT Delhi - RAM Maths Circle
Session 19

(Organized by the Department of Mathematics, IIIT Delhi)

IIIT-Delhi

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1. Find all solutions in positive integers to the equation

$$m^4 = n^3 + 137.$$

2. If $a < b < c < d$ are distinct positive integers such that $a + b + c + d$ is a square, what is the minimum value of $c + d$?

3. Find the remainder when

$$10^{20} + 10^{21} + \dots + 10^{2021}$$

is divided by

$$\underbrace{44 \cdots 4}_{\text{a string of 4s}}.$$

4. Without a calculator, find a factor of

$$8^{59} - 2^{19} + 6^9$$

that lies between 2000 and 3000.

5. Find all polynomials f that satisfy the equation

$$\frac{f(9x)}{f(3x)} = \frac{243x - 729}{x - 81}$$

for infinitely many values of x .

Counting: Partitions

We may already know how to solve problems such as the following:

For how many triplets (x, y, z) can we write

$$x + y + z = 7?$$

This week, we will ask some other questions along similar lines. Try the following:

For how many triplets (x, y, z) can we write

$$x + y + z = 7,$$

with the additional condition that $x \geq y \geq z$?

As you can see, the answer to this new problem will be different from the answer to the first one. We will call these ordered collections of numbers that add up to a given number *partitions*.

For example, since

$$5 + 4 + 3 = 12,$$

we say that $(5, 4, 3)$ is a partition of 12. We can represent this partition visually by drawing 5 squares above 4 squares above 3 squares, all aligned, as shown in the picture on the board.

Exercises.

- Draw block diagrams for all of the partitions of 7.
- Draw the partitions $(4, 4, 3, 1, 1)$ and $(5, 3, 3, 2)$.
- What relationship do you observe between these two drawings?
- What number does $(4, 4, 3, 1, 1)$ partition? What number does $(5, 3, 3, 2)$ partition?
- Draw the partitions $(3, 3, 1)$ and $(2, 1, 1)$. Can you see a relationship between the drawings?

We will call a partition *self-conjugate* if, when its diagram is flipped (as above), we obtain the same diagram we started with.

- Draw some examples of self-conjugate partitions.