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## IIIT Delhi - RAM Maths Circle Session 20

(Organized by the Department of Mathematics, IIIT Delhi)

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The triangle inequality is easily motivated, whether or not students have had a formal introduction to geometry. But even for those students who have studied axiomatics or formal proof, there are non-trivial applications lying right beneath the surface, and problems involving the triangle inequality can be constructed which demand significant thought.

The inequality itself states that for any triangle  $ABC$  we have three inequalities showing that any side of the triangle is less than the sum of the other two:

$$AB < AC + BC, \quad AC < AB + BC, \quad BC < AB + AC.$$

**Problem 1.** A mushroom-gatherer leaves the woods at a given point. He must reach a highway, which follows a straight line, and re-enter the woods at another given point. How should he do this following the shortest path?

**Problem 2.** A woodsman's hut is inside a peninsula in the shape of an acute angle. The woodsman must leave his hut, walk to one shore, then to the other shore, and return home. How should he choose the shortest such path?

**Problem 3.** Point  $A$ , inside an acute angle, is reflected in each side of the angle to obtain points  $B$  and  $C$ . Segment  $BC$  intersects the sides of the angle at  $D$  and  $E$ . Show that

$$\frac{BC}{2} > DE.$$

**Problem 4.** Point  $C$  lies inside a given right angle, and points  $A$  and  $B$  lie on its sides. Prove that the perimeter of triangle  $ABC$  is not less than twice the distance  $OC$ , where  $O$  is the vertex of the right angle. greater than half the perimeter of the triangle.

**Problem 5.** If point  $O$  is inside triangle  $ABC$ , prove that

$$AO + OC < AB + BC.$$

**Problem 6.** Prove that the sum of the distances from a point to the vertices of a triangle is less than the perimeter if the point lies inside the triangle. What if the point lies outside?

**Problem 7.** Solve Problem 11 if the peninsula has the shape of an obtuse angle.

**Problem 8.** Prove that the length of median  $AM$  in triangle  $ABC$  is not greater than half the sum of sides  $AB$  and  $AC$ . Prove also that the sum of the lengths of the three medians is not greater than the perimeter.

**Problem 9.** A polygon is cut out of paper and folded along a straight line. Prove that the perimeter of the folded polygon is not greater than the original perimeter.

**Problem 10.** Prove that a convex polygon cannot have three sides, each longer than the longest diagonal.

**Problem 11.** Prove that the perimeter of a triangle is not greater than  $\frac{4}{3}$  the sum of its medians.

**Problem 12.** Two villages lie on opposite sides of a river whose banks are parallel lines. A bridge is to be built perpendicular to the banks. Where should the bridge be built so that the path between the villages is shortest?

**Problem 13.** Prove that a convex pentagon has three diagonals which can form a triangle.

