RAM Maths Circle

January 25 2026

Nagpur

Introduction

Today's session began with digit-based experiments that revealed consistent outcomes despite varied starting choices, encouraging students to identify hidden structure and logical regularities. Building on these observations, the session progressed to greedy strategies using Egyptian fractions. Students examined how locally optimal decisions affect overall outcomes, connecting historical mathematical methods with modern problem-solving and algorithmic reasoning.

Warm-Up Activities

Activity 1: Reverse and Subtract

Students began with an exploratory number activity to observe hidden patterns.

Task:

- Choose a 2-digit, 3-digit, or 4-digit number with all digits distinct.
- Reverse the digits to form a new number.
- Subtract the smaller number from the larger one.

Observation:

- For 2-digit numbers, the result is always a multiple of 9.
- For 3-digit numbers, the result is always a multiple of 99. You get 495 at some point
- For 4-digit numbers, the result is always a multiple of 999.You get 4995 at some point

Discussion Focus: Students discussed why digit reversal and subtraction consistently produce multiples of these numbers, leading to the idea of structural invariants in number systems.

Activity 2: Kaprekar's Constant (6174)

This activity introduced students to an iterative numerical process with a surprising fixed outcome.

Task:

- Choose any 4-digit number with all digits distinct.
- Arrange the digits in descending and ascending order.
- Subtract the smaller number from the larger one.
- Repeat the process with the new result.

Key Result: Students observed that, in at most seven steps, the process always reaches the number

$$6174$$

known as *Kaprekar's constant*.

Exploration: Egyptian Fractions and Greedy Strategies

The ancient Egyptians represented fractions as sums of unit fractions (fractions with numerator 1), with no denominator allowed to repeat.

Problem. Write the fraction $\frac{3}{5}$ as a sum of *distinct unit fractions*.

Student Instructions:

- Break $\frac{3}{5}$ into fractions of the form $\frac{1}{n}$.
- Use more than one term if needed.
- Do not repeat denominators.

Discussion Focus: Students examined why attempts such as

$$\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

fail, highlighting the importance of distinct denominators.

Strategy Discovery: The Greedy Idea

Through guided discussion, students observed the following strategy:

Always subtract the largest possible unit fraction that does not exceed the given fraction.

Worked Example:

- The largest unit fraction $\leq \frac{3}{5}$ is $\frac{1}{2}$.

- Subtracting gives $\frac{3}{5} - \frac{1}{2} = \frac{1}{10}$.

$$\frac{3}{5} = \frac{1}{2} + \frac{1}{10}$$

Reflection Prompt: Why is choosing $\frac{1}{2}$ more effective than starting with $\frac{1}{3}$?

Multi-Step Greedy Exploration

Problem. Apply the same idea to write $\frac{2}{7}$ as a sum of distinct unit fractions.

Student Task:

- Repeat the greedy subtraction process.
- Continue until all remaining terms are unit fractions.

Solution:

$$\frac{2}{7} = \frac{1}{4} + \frac{1}{28}$$

Key Insight: Greedy strategies may require multiple steps and intermediate fractions before completion.

The Large Denominator Trap

Problem. Apply the greedy method to $\frac{4}{13}$.

$$\frac{4}{13} = \frac{1}{4} + \frac{1}{18} + \frac{1}{468}$$

Discussion Focus: Students discussed why a simple fraction can lead to very large denominators, illustrating a limitation of greedy methods.

Structural Identities and Non-Uniqueness

Students explored whether Egyptian fraction representations are unique.

Identity:

$$\frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)}$$

Implications:

- Any unit fraction can be split into smaller unit fractions.
- Egyptian fraction representations are not unique.

Example:

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$$

Student Task: Starting from $\frac{1}{2}$, construct an Egyptian fraction representation using at least four unit fractions.

Conclusion

Through experimentation and discussion, students gained insight into greedy algorithms, their strengths, and their limitations. The lesson connected historical mathematics with modern algorithmic thinking, reinforcing logical reasoning and structured problem-solving skills.