RAM Math Circle - Chennai

Synopsis for August 10 2025

We explored A few problems that will lead us to the concept of degree of a vertex in graph theory (for next time). The problems combine combinatorial thinking with graph techniques.

The first four problems are a good example of how the same underlying problem can appear in different situations.

1. There are 9 computers connected with several cables to make a network (each cable connects a pair of machines). Is it possible for each computer to be connected to exactly 3 others?
2. In the country of Farawaynia, there are 9 towns; some are connected by roads, others are not. Is it possible that each of the towns is connected to exactly 3 other towns?
3. In a certain company, each employee has exactly 3 friends among the co-workers. Is it possible that this company has exactly 9 employees?
4. It is known that 3 edges meet at every vertex of a polyhedron. Is it possible for this polyhedron to have exactly 9 vertices?
5. Twelve gentlemen met at a part. 10 of them shook hands with everyone else; the other 2 (who quarreled) shook hands with everyone but each other. How many handshakes took place?
6. Our office has 9 computers.
   (a) Link them so every computer is connected with exactly 4 others. How many cables are needed? Can you draw such a configuration?
   (b) Connect each computer with exactly 5 others. Is this possible?

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For the next part, we explored the following exercises in geometry with the aid of paper folding. This set of exercises is an adaptation of the chapter on equilateral triangles in the book 'Geometric exercises in paper folding' by T. Sundara Row.

1. Begin with your square piece of paper. Fold one pair of opposite edges together so they match perfectly. Unfold.
   - What shape does the crease make?
   - Mark the midpoint of each side touched by the crease.
2. Pick any point along the crease. Fold a crease through this point and the two corners of the square that are on each side of it.
   - Notice the triangle shape that forms at the fold.
   - Draw or outline the triangle formed.
   - How does the crease divide this triangle?
3. If you choose any point C on the fold crease and form a triangle CAB, the resulting triangle will be ______________________.
4. Now if we choose C so that $CA = AB$, then the resulting triangle will be ______________________.
5. How do we choose a point C so as to give us an equilateral triangle ABC?
6. Can you identify the folds that will give you the altitudes in ΔABC?

7. Fold each corner of the triangle to the midpoint of the opposite side (the base). Unfold to see the new creases.
   - How many creases do you see?
   - Draw the triangle and all its creases. Label them as altitudes.
8. Each altitude splits the triangle into two equal right-angled triangles. Shade all six right triangles you find.
9. Notice where all three altitudes cross—mark that point O.
   - Is this point inside the triangle?
   - Why do you think it's special?
10. Use a compass or trace to draw a circle centered at point O that passes through points A, B, and C. Draw another circle centered at O that just touches all three sides (inscribed circle).
11. Compare the two circles. Measure their radii. What is the relationship between the radii of the big circle and the small circle?
12. Look at the angles made by the altitudes at point O.
   - How many equal angles meet at the center? ____________
   - What is the measure of each angle? ____________
13. Find the measure of the angles at each corner of your equilateral triangle.
14. Draw and fold the triangle inside your triangle (connect the midpoints of each side with straight lines to form a smaller triangle).

    How does the size compare with the original?
15. Find and draw any rhombuses formed by the creases and folds. Color these shapes.
16. How many congruent right-angled triangles can you spot in your folded triangle?
17. Are the rhombuses all the same size? Mark and compare them.

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Problems for practice

1. During a chess tournament, some people played 5 games, some played 6. Prove that the number of people who played 5 games is even.
2. Eight people came to a party; some shook hands. Is it possible that 6 of them shook hands with 6 different people, and 2 shook hands with only 2 each?
3. Martian amoebas: a red amoeba divides into 5 blue amoebas; a blue into 7 red. Starting with 1, you finish with 100 in the jar. Why must some have escaped?

Cup Game:

You are given 7 cups placed horizontally, with 3 of them kept upside down (open side down) and 4 kept right side up (bottom side down). Which 3 are upside down and which 4 are right side up does not matter. Nevertheless, let us assume the first 3 cups from the right-hand side are upside down.

A move in the game is defined as follows: take any two cups and flip both of their orientations, i.e., if both were up, turn them down; if both were down, turn them up; and if one was up and the other down, flip each (the up one becomes down, and the down one becomes up).

Question: After any number of such moves, is there a way to make all the cups face up?

Students realized that this is not possible, as the parity of the number of cups facing up and the number of cups facing down remains invariant throughout the game. Each move involves flipping two cups, which either changes both from the same orientation to the opposite (leaving the total number of ups and downs unchanged), or swaps one up and one down (again preserving the counts). Therefore, if we start with an odd number of cups facing down, we can never reach a state where all cups are facing up.

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