

RAM Maths Circle

Date: October 12, 2025
Krea University - TTK Road

Continues discussion on Sudoku and Euler's problem. Introduces thinking about permutations and the permutation group.

1 Sudoku

1. Why is it ok to start filling in the blank Sudoku with 123456789 on the top row?
2. What are possible choices for other groups as you go further?
3. What transformations of a solved Sudoku board yield other solutions?
4. What is perhaps the complete set of such transformations?
5. If we consider two solutions to be "equivalent" if they one can be transformed into the other, then how does this affect the total number of solutions of the Sudoku board?
6. Are all solutions of the Sudoku board transformable between each other? Can you construct a counter example?
7. If $C_{(mn)}$ represents swapping the columns m and n and similarly $R_{(mn)}$ for the rows, how can we write all the solution preserving swappings? Furthermore, if P_{mn} represents exchanging m and n wherever they occur on the board, can we construct any ordering of 123456789 by applying a sequence of P_{mn} operations?
8. Can we construct any ordering of the columns using an appropriate sequence of $C_{(mn)}$?
9. Does the ordering matter for doing a sequence of these swappings? For which swappings does the ordering matter and are there any for which the ordering doesn't matter? For example is $C_{mn}R_{jk} = R_{jk}C_{mn}$?

10. Is the set of $C_{(mn)}$ and $R_{(mn)}$ and combinations of them sufficient to describe all solution-preserving transformations? They certainly cover reflections $C_{19}C_{28}C_{37}C_{46}$ and correspondingly for rows. Can you express rotations also with them? What about diagonal reflections? Will your answers differ if the board were filled with numbers $1 \cdots 81$ instead of the usual Sudoku set of numbers $9 \times [1, 9]$?

Notation: Considering permutations of a sequence, for example, 1234, we write the permutation to 4123 as (1234). This is called a "cyclic permutation". So a swap of positions 2 and 4 in 1234 is written as (24).

2 Euler's problem

1. Is there any similarity with the Sudoku board?
2. Solve for n times n where n is odd - i.e. Give a method for solving it that will work for all odd n .
3. Do you have such a strategy that also works for even n ?
4. Which ones did you manage to solve and which ones did you have difficulty with?
5. What were your approaches/strategies for solving it for various n .

