# RAM Maths Circle

## July 13, 2025

Nagpur

This session introduced students to key number theory concepts through divisibility and problem-solving. The students were introduced to the concept of parity of numbers which shows if the number is even or odd .Topics like factors, multiples, and prime numbers were explored, along with rules of divisibility and pattern recognition. The session aimed to strengthen number sense and encourage logical thinking, laying a strong foundation for future mathematical learning.

## Questions

## Problem 1: The 100 Doors Problem

There are 100 doors, all initially closed.

One by one, 100 students walk by the doors:

- The 1st student toggles **every door** (i.e., opens all doors).
- The 2nd student toggles **every 2nd door** (i.e., doors 2, 4, 6, . . . ).
- The 3rd student toggles **every 3rd door** (i.e., doors 3, 6, 9, . . . ).
- ⋮
- The 100th student toggles **only the 100th door**.

After all 100 students have passed, which doors will remain **open** and which will be **closed**?

Can you find a **pattern or rule** to determine which door numbers stay open? The main highlight here is on the parity of a number. The students were asked to check if door number 2048 would be opened or closed.

# Problem 2: The Divisibility Challenge

A teacher wrote a large number on the board and asked the students to mention one divisor of the number each, in order.

The 1st student said, "The number is divisible by 2."
The 2nd student said, "The number is divisible by 3."
The 3rd student said, "The number is divisible by 4."
⋮
The 30th student said, "The number is divisible by 31."

The teacher then commented that "exactly two students, who spoke consecutively, were wrong" — all others were correct.

**Question:** Which two students were wrong?

Adding to that, Students were introduced to 2-11 divisibility tests, and they didn't know the divisibility test of 7, so they were amazed by it.

# Task for Students

**Problem 1. 6-Digit Security Code with Divisibility Constraints**

There is a 6-digit security code formed using the digits from 1 to 6, with **no repetition allowed**.

The code must satisfy the following conditions:

- The number formed by the **first 2 digits** must be divisible by 2.
- The number formed by the **first 3 digits** must be divisible by 3.
- The number formed by the **first 4 digits** must be divisible by 4.
- The number formed by the **first 5 digits** must be divisible by 5.
- The number formed by **all 6 digits** must be divisible by 6.

Identify the 6-digit number that satisfies all the above conditions. Also, investigate whether other such numbers meet the requirements.

**Problem 2. Explore Divisibility Rule Patterns**

"It was observed that the divisibility rules for certain numbers resemble those of their lower powers."

- Powers of 2: **2, 4, 8, . . .**
- Powers of 5: **5, 25, . . .**

The students were asked to explore this pattern further to figure out why this works and to **investigate whether the divisibility rule for 3 and 9 can be extended to 27** using a similar logic.