RAM Math Circle - Chennai

Synopsis for November 09 2025

This Math Circle session was conducted in a small-group discussion format. Students were divided into three groups, each working in a separate space. One group focused on number theory, while the other two groups worked on combinatorics. Throughout the session, students were encouraged to work collaboratively and explain their reasoning to their friends. Volunteers played an active role in facilitating discussion, asking probing questions, and ensuring that all students were engaged. This structure promoted mathematical communication, coming up with problem-solving strategies, and confidence in presenting ideas. This session report highlights the main ideas and discussion themes from the session, for the complete list of problems, please refer to the RAM Math Circle worksheet dated November 9, 2025.

Combinatorics Group: A

The combinatorics group revisited ideas that had been introduced earlier in the semester. Students recalled the binomial coefficient

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!},$$

and discussed its interpretation as the number of ways of choosing r objects from a set of n distinct objects, where the order does not matter. Care was taken to distinguish between *selection* and *arrangement*. Students also discussed why the expression only makes sense for $0 \leq r \leq n$.

Pascal's Triangle

Students constructed rows of Pascal's triangle and explored its structure. Two key properties were discussed in detail:

- **Mirror symmetry:** why $\binom{n}{i} = \binom{n}{n-i}$.
- **Pascal's rule:**

$$\binom{n}{i} = \binom{n-1}{i-1} + \binom{n-1}{i}.$$

Both algebraic and combinatorial proofs of Pascal's rule were explored. In the combinatorial proof, students analysed a selection problem by splitting into cases depending on whether a specific person was included or excluded. This helped them see how counting arguments naturally give rise to algebraic identities. Students worked on problems such as

- Counting teams or committees under given conditions (for example, "at least one particular person must be included").
- Explaining why choosing a subset is equivalent to choosing its complement.
- Comparing two different counting orders and showing that they give the same result.

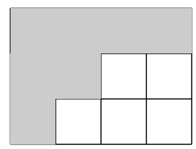
Combinatorics Group: B

The second combinatorics group worked on a problem involving partitions inside a rectangular grid. Students were first introduced to the idea of a *shaded region* or a *valid region* inside an $m \times n$ rectangle, with the condition that the shaded region should be left-justified and should not have any gaps.

Through examples and discussion, students observed that any such valid shaded region can be interpreted as a partition fitting inside the rectangle. This helped them connect the problem to familiar ideas from earlier sessions. We considered a 3×4 rectangle. A valid shaded region can be described by specifying how many boxes are shaded in each row, starting from the top, with the conditions that:

- each row has at most 4 boxes, and
- the number of shaded boxes does not increase as we move down the rows.

For example, the choice $(4, 2, 1)$ represents a partition with 4 boxes in the first row, 2 in the second row, and 1 in the third row.



Example partition $(4, 2, 1)$ inside a 3×4 rectangle

Students then associated each valid shaded region with a monotone lattice path from the top-left corner to the bottom-right corner of the rectangle, using only right and down steps. Each such path corresponds to exactly one shaded region.

Since a path consists of 3 downward steps and 4 rightward steps, the total number of such paths is

$$\binom{3+4}{3} = \binom{7}{3}.$$

Thus, the number of partitions fitting inside a 3×4 rectangle is given by a binomial coefficient.

The group then explored the partial order on these partitions, where one partition is said to be less than another if its shaded region is contained inside the other. This containment relation defines a poset.

Students observed that the total number of shaded boxes gives a natural notion of *rank*. In the 3×4 case, the minimum rank is 0 (no boxes shaded) and the maximum rank is 12 (all boxes shaded).

Using diagrams and counting arguments, students noticed a symmetry: the number of partitions with k boxes is the same as the number of partitions with $12 - k$ boxes. This rank symmetry was discussed both visually and conceptually using the lattice-path interpretation.

Number Theory Group

The number theory group focused on modular arithmetic and number representations in different bases.

Modular Arithmetic

Students recalled the division algorithm and discussed why remainders modulo a number must lie in a fixed range. Modular notation was introduced, and students computed examples involving addition, multiplication, and powers modulo a prime. Group discussion focused on how modular arithmetic behaves with respect to ordinary arithmetic operations.

Number Bases

Students explored number systems other than base 10, including base 2, base 3, base 5, and base 16. They practised:

- Reading numbers written in base p by expanding them using powers of p.
- Converting numbers from base 10 to base p using repeated division.
- Identifying invalid representations and common mistakes.

Extension problems were provided for students who finished early, including questions involving hexadecimal representations.