

## RAM - Maths Circle

Date: November 23, 2025  
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A medley of problems, taken from a number of other math circles.

### 1. Problems

1. Let  $p$  and  $q$  be prime numbers and  $n$  be a whole number such that

$$\frac{1}{p} + \frac{1}{q} + \frac{1}{pq} = \frac{1}{n}$$

. Find all possible sets of values for  $p$ ,  $q$  and  $n$ .

2. The increasing sequence 1, 3, 4, 9, 10, 12, 13 consists of all those positive integers which are powers of 3 or sums of distinct powers of 3 (including  $3^0$ ). Find the 100<sup>th</sup> term of the sequence.
3. Assume that  $a$ ,  $b$ ,  $c$  and  $d$  are positive integers such that  $a^5 = b^4$ ,  $c^3 = d^2$ , and  $c - a = 19$ . Find  $d - b$ .
4. Suppose that points  $A$ ,  $B$ ,  $C$  are vertices of a scalene triangle. How many points  $D$  in the plane of  $\triangle ABC$  have the property that the quadrilateral  $ABCD$  has at least one axis of symmetry?
5. By a proper divisor of a natural number we mean a positive integral divisor other than 1 and the number itself. A natural number greater than 1 will be called nice if it is equal to the product of its distinct proper divisors. What is the sum of the first ten nice numbers?
6. A beaker filled to the brim with water weighs 500 grams, while the same beaker filled halfway weighs 325 grams. How many grams of water can the beaker hold?
7. A train moves in one direction for 5.5 hours. If the train covers **any** 100 km segment in 1 hour:  
(a) Is the train necessarily moving at a constant rate? (b) Is the train's average speed necessarily 100 kmph?
8. Let  $n$  be a positive integer. Show that the set of weights

$$1, 3, 3^2, 3^3, 3^4, \dots$$

can be used to weigh an  $n$ -gram weight (using both pans of the scale). Show that this can be done in exactly one way.

9. How many terminal zeroes are there if you expand  $1000!$  completely.

$$1000! = 1 \times 2 \times 3 \times \dots \times 1000$$

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10. Does there exist a triangle such that the sum of two of its altitudes is larger than the sum of the two corresponding bases?