RAM Maths Circle

Date: September 14, 2025

IIITD Campus, Delhi

1 Introduction

Our goal is to ignite interest and foster a love for problem-solving among the students. The parents interacted with the faculty and past alumni and were introduced to the great benefits of continuing with this circle, and to other RAM programs.

2 The Birthday Paradox

The birthday paradox asks: "In a room of people, how many individuals do we need before there's a greater than 50% chance that two people share the same birthday?" Most people intuitively guess numbers like 100 or 180, reasoning that with 365 possible birthdays, you'd need roughly half that many people. However, the actual answer is just 23 people.

This session was taught by Prof. Antar Bandyopadhyay, who is a Prof. at the Theoretical Statistics and Mathematics Division of the Indian Statistical Institute (ISI). Prof. Bandyopadhyay explained how we calculate the probability by considering the complement: the chance that no two people share a birthday. With each additional person, the probability of avoiding birthday matches decreases rapidly, which leads to the surprising needing just 23 people for a greater than 50% probability of a match.

3 Problem 1: Powers of 10 and Divisibility by 3

Problem Statement: Compute the remainders when the positive powers of 10: 10^n, $n \geq 0$, that is, $10^0, 10^1, 10^2, 10^3, \ldots$ are divided by 3. Do you observe any patterns or repetitions in the remainders? Based on your observation, propose a divisibility test for determining whether a number is divisible by 3.

3.1 Observation

Students began by calculating the remainders of successive powers of 10 when divided by 3:

- $10^0 = 1$ remainder 1 when divided by 3
- $10^1 = 10$ remainder 1 when divided by 3

- $10^2 = 100$ remainder 1 when divided by 3
- $10^3 = 1000$ remainder 1 when divided by 3
- $10^4 = 10000$ remainder 1 when divided by 3

Observation: dividing any whole number power of 10 by 3 always leaves a remainder of 1. This consistent pattern holds for all non-negative integer powers of 10.

3.2 Divisibility Test

Building on their observation, students explored how this pattern connects to the familiar divisibility rule for 3. We know that a number is divisible by 3 if and only if the sum of its digits is divisible by 3. Using the observation that each power of 10 leaves remainder 1 when divided by 3, students worked to understand why this rule works.

Any number can be written in expanded form. For example, the number 2347 can be expressed as:

$$2347 = 2 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

Since each power of 10 is congruent to 1 modulo 3, we have:

$$2347 \equiv 2 \times 1 + 3 \times 1 + 4 \times 1 + 7 \times 1 \equiv 2 + 3 + 4 + 7 \pmod 3$$

This demonstrates that any number has the same remainder when divided by 3 as the sum of its digits. Therefore, a number is divisible by 3 if and only if the sum of its digits is divisible by 3.

3.3 Student Presentations

After the problem-solving time elapsed, students came to the front and explained their innovative approaches to everyone at the whiteboard. Several students demonstrated different methods of discovering the pattern. These presentations allowed students to see that there are often various valid approaches to solving mathematical problems.