RAM Maths Circle
Feb. 16, 2025
Nagpur

Problem 1

Solve for whole numbers A, B, C:

$$A \times B \times C = 240 \ A \times C$$
$$+ B = 46 \ A + B \times C$$
$$= 64$$

Find $A + B + C$.

Solution:

We start by factoring 240 into possible values for A, B, C:

$$240 = 2^4 \times 3 \times 5$$

Possible integer values satisfying $A \times B \times C = 240$ are examined. We check suitable triplets for the other equations:
   Assuming $A = 4$, we solve for B and C:

$$4 \times C + B = 46$$
$$4 + B \times C = 64$$

Substituting $C = 10$ from trials:

$$4 \times 10 + B = 46$$
$$B = 6$$

$$4 + 6 \times 10 = 64$$

  All conditions hold, so:

$$A + B + C = 4 + 6 + 10 = \mathbf{20}$$

Problem 2

Given:

$$a + 1 = b + 2 = c + 3 = d + 4 = e + 5 = a + b + c + d + e + 3$$

Find $a^2 + b^2 + c^2 + d^2 + e^2$.

Solution:
Let the common value be k:

$$k = a + 1 = b + 2 = c + 3 = d + 4 = e + 5 = a + b + c + d + e + 3$$

Substituting into k:

$$k = (k - 1) + (k - 2) + (k - 3) + (k - 4) + (k - 5) + 3$$

Solving:

$$k = 5k - 15 + 3$$

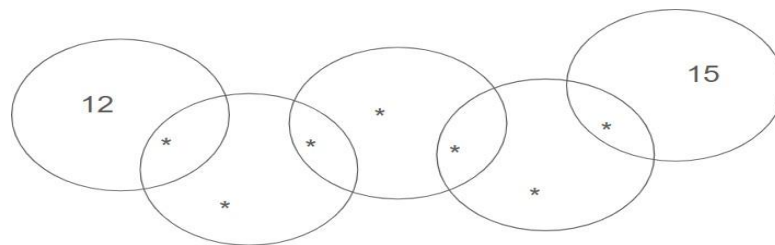$$k - 5k = -12$$

$$-4k = -12$$

$$k = 3$$

Thus,

$$a = 2, \ b = 1, \ c = 0, \ d = -1, \ e = -2$$

Computing the sum of squares:

$$a^2 + b^2 + c^2 + d^2 + e^2 = 2^2 + 1^2 + 0^2 + (-1)^2 + (-2)^2$$
$$= 4 + 1 + 0 + 1 + 4 = \mathbf{10}$$

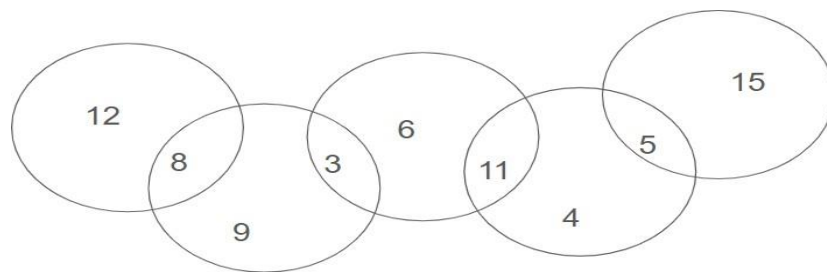## Problem 3

Assign numbers {3,4,5,6,8,9,11} exactly in place of * so that the sum of each number in the circle is 20. Is the solution unique?

Solution:

One possible solution is:

Students explored more varieties of combinations to determine whether the solution is unique.