

RAM Maths Circle

March 14, 2025

Nagpur

Introduction

In this session, participants explored the deeper structure of complex numbers through pattern-based reasoning. They discovered how the powers of 'i' repeat cyclically and how modulus and conjugates reveal hidden symmetry. We also introduced roots of unity to show how complex numbers arrange themselves in geometric patterns.

The goal was to help students see complex numbers not just as symbols, but as a structured system with predictable behavior. By the end, they could interpret, calculate, and connect these concepts with clarity and confidence.

Exploration

It was awesome watching students navigate imaginary numbers like a puzzle with curiosity and confidence. Initially confused, they turned the whole topic into a fun challenge. By the end, they were actually enjoying complex numbers, which is amazing.

QUESTION-1

Problem: Evaluate $i^1 + i^2 + i^4 + i^6 + \dots + i^{2n}$

QUESTION-2

Problem: Evaluate : $i^n + i^{n+1} + i^{n+2} + i^{n+3}$.

QUESTION-3

Problem: Evaluate: $\sum_{n=1}^{13} (i^n + i^{n+1})$.

QUESTION-4

Problem: For a positive integer n , find the value of $(1-i)^n \left(1 - \frac{1}{i}\right)^n$.

QUESTION-5

Problem: If $x = -5 + 2\sqrt{4}$, find the value of $x^4 + 9x^3 + 35x^2 - x + 4$.

QUESTION-6

Problem: Find the value of $x^3 + 7x^2 - x + 16$, when $x = 1 + 2i$.

QUESTION-7

Problem: Find the value of: $2x^4 + 5x^3 + 7x^2 - x + 41$, where $x = -2 - 3i$

QUESTION-8

Problem: Find the value of $2x^4 + 5x^3 + 7x^2 - x + 41$, where $x = -2 - \sqrt{3}i$.

QUESTION-9

Problem: Find the square root of the following complex numbers: (i) $7 - 4i$ (ii) $5 + 2i$ (iii) $-15 - 8i$ (iv) i (v) $-7 - 24i$ (vi) $-5 + 12i$

QUESTION-10

Problem: Find the value of $\omega^4 = \omega$.

QUESTION-11

Problem: Find the value of $\frac{1}{1-\omega} + \frac{1}{1-\omega^2}$.

QUESTION-12

Problem: Find the value of $\omega^5 + \omega^6 + \omega^7$.

QUESTION-13

Problem: Find the value of $(1 + \omega)^2 + (1 + \omega^2)^2$.

