At first, we should solve the problems from the previous session (session 18) where we discussed sets. Then we will try these new problems.

Cartesian Product of sets

Let A and B be two sets. The *Cartesian product* of A and B is

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}.$$

The pair (a, b) is called an *ordered tuple*. By ordered, we mean $(a, b) \neq (b, a)$ if $a \neq b$. For example, if $A = \{0, 1, 2\}$ and $B = \{0, a\}$, then

$$A \times B = \{(0, 0), (0, a), (1, 0), (1, a), (2, 0), (2, a)\}.$$

We have seen that two sets A and B will be equal if $A \subseteq B$ and $B \subseteq A$, that is, all elements of A are in B and all elements of B are in A.

Problems

Problem 1: Find the cartesian product $A \times B$ and $B \times A$ for the following sets. Also for each find the number of elements in the cartesian products.

   (i) $A = \{0, 1\}$ and $B = \{0, 1\}$.

  (ii) $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$.

 (iii) $A = \varnothing$ and $B = \{1, 2, 3\}$.

Problem 2: Can you generalize the number from problem 1. That is, if $|A| = m$ and $|B| = n$, then find out $|A \times B|$ and $|B \times A|$, where $|A|$ denotes the number of elements in the set A.

Problem 3: Suppose $|A| = 5$ and $|A \times B| = 200$, then can you find $|B|$?

Problem 4: If $A \subseteq B$, then for any set C is it always true that $A \times C \subseteq B \times C$? If $A \subseteq B$ and $C \subseteq D$ then what can you say about $A \times C$ and $B \times D$?

Problem 5: Let A and B both are nonempty sets. If $A \times B = B \times A$, then can you show that $A = B$ (note that in order to show that $A = B$, you need to show $A \subseteq B$ and $b \subseteq A$)?

Problem 6: Let $X = \{1, 2, 3\}$, $A = \{1, 2\}$ and $B = \{2, 3\}$.

(i) Find $A \times B$.

(ii) Find $(A \times B)^c$.

(iii) Find $A^c \times B^c$.

(iv) Are $(A \times B)^c$ and $A^c \times B^c$ equal? Is $A^c \times B^c \subseteq (A \times B)^c$?

Problem 7:

(i) Given any sets A and B, prove that $A^c \times B^c \subseteq (A \times B)^c$.

(ii) Show that $(A \times B)^c = (A^c \times B) \cup (A \times B^c) \times (A^c \times B^c)$.

End of problem sheet