

RAM Math Circle - Chennai
Synopsis for July 27 2025

Today's session was aimed at recalling the counting strategies we have studied so far, learning about permutations and combinations and practicing these strategies.

1 Recall

So far we have seen 3 strategies for counting:

- The addition principle
- The multiplication principle
- Number of arrangements of n objects taken r at a time, with repetition allowed.

Let's recall each of these with simple examples:

1. Given 3 different math books, 5 different history books and 2 different geography books, how many ways are there to select 1 book?

Answer:

2. Given 3 different math books, 5 different history books and 2 different geography books, how many ways are there to choose 3 books so you have 1 book of each subject?

Answer:

3. How many binary sequences of length n are there?

Answer:

2 Two more strategies

A **permutation** (or r -permutation) of a set S containing n different objects is an ordered arrangement of r of the n elements of S in a row. The number of such permutations is given by

$$r = \frac{n!}{(n-r)!}$$

1. Consider 4-digit numbers whose digits are taken from the set $S = \{1, 2, 3, 4, 5\}$. Count all 4-digit numbers that satisfy the condition
 - (a) the digits are distinct
 - (b) the number is even
 - (c) both the above conditions

2. In how many ways can 10 examination papers be set so that no two of the three mathematical papers are consecutive?

A **combination** (or r -combination) of a set S containing n different objects is an un-ordered selection of r of the n elements of S . The number of such combinations is given by

$$r = \frac{n!}{r!(n-r)!}$$

1. In how many ways can a team be formed with 4 elves and 6 goblins with at least 2 elves and twice as many goblins?

2. In a bag of 15 tennis balls there are 7 white ones. In how many ways can 12 balls be selected so as to include

- (a) exactly 6 white balls?
- (b) at least 6 white balls?

3 Practice set

1. In how many ways can all the cards be arranged from a 52-card deck?
2. How many different 10-letter 'words' (sequences) are there with no repeated letters, formed from the 26-letter English alphabet?
3. How many ways are there to distribute 9 different books among 15 children if no child gets more than 1 book?
4. Suppose 5 guests A, B, C, D, E are to speak at a conference. In how many ways can the organisers order the talks so that
 - (a) B does not speak before A ?
 - (b) A speaks immediately before B ?
5. How many ways are there to pick a 5-person team from 10 possible players? How many teams if the strongest player and the weakest player must be on the team?
6. In how many ways can 12 identical white and 12 identical black pawns be placed on a the black squares of an 8×8 chessboard?
7. Find the sum of all 3-digit numbers that can be formed using the digits 1, 2, 3, 4.
8. Find the sum of all 5-digit numbers, with distinct digits, that can be formed using the digits 1, 2, 3, 4, 5.

