RAM Math Circle - Chennai Synopsis for July 27 2025

Today's session was aimed at recalling the counting strategies we have studied so far, learning about permutations and combinations and practicing these strategies.

1 Recall

So far we have seen 3 strategies for counting:

- The addition principle
- The multiplication principle
- \bullet Number of arrangements of n objects taken r at a time, with repetition allowed.

Let's recall each of these with simple examples:

1. Given 3 different math books, 5 different history books and 2 different geography books, how many ways are there to select 1 book?

Answer:

- 2. Given 3 different math books, 5 different history books and 2 different geography books, how many ways are there to choose 3 books so you have 1 book of each subject?

 Answer:
- 3. How many binary sequences of length n are there? Answer:

2 Two more strategies

A permutation (or r-permutation) of a set S containing n different objects is an ordered arrangement of r of the n elements of S in a row. The number of such permutations is given by

$$r = \frac{n!}{(n-r)!}$$

1. Consider 4-digit numbers whose digits are taken from the set $S = \{1, 2, 3, 4, 5\}$. Count all 4-digit numbers that satisfy the condition

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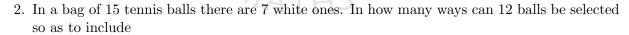
- (a) the digits are distinct
- (b) the number is even
- (c) both the above conditions

2. In how many ways can 10 examination papers be set so that no two of the three mathematical papers are consecutive?

A combination (or r-combination) of a set S containing n different objects is an un-ordered selection of r of the n elements of S. The number of such combinations is given by

$$r = \frac{n!}{r!(n-r)!}$$

1. In how many ways can a team be formed with 4 elves and 6 goblins with at least 2 elves and twice as many goblins?



- (a) exactly 6 white balls?
- (b) at least 6 white balls?

3 Practice set

- 1. In how many ways can all the cards be arranged from a 52-card deck?
- 2. How many different 10-letter 'words' (sequences) are there with no repeated letters, formed from the 26-letter English alphabet?
- 3. How many ways are there to distribute 9 different books among 15 children if no child gets more than 1 book?
- 4. Suppose 5 guests A, B, C, D, E are to speak at a conference. In how many ways can the organisers order the talks so that
 - (a) B does not speak before A?
 - (b) A speaks immediately before B?
- 5. How many ways are there to pick a 5—person team from 10 possible players? How many teams if the strongest player and the weakest player must be on the team?
- 6. In how many ways can 12 identical white and 12 identical black pawns be placed on a the black squares of an 8×8 chessboard?
- 7. Find the sum of all 3-digit numbers that can be formed using the digits 1, 2, 3, 4.

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8. Find the sum of all 5-digit numbers, with distinct digits, that can be formed using the digits 1, 2, 3, 4, 5.