1 Junior batch

The session started with a set of warm-up problems intended to activate students' existing intuition about factors and divisibility before introducing the formal definition.

1. Is $2^9 \cdot 3$ divisible by 2? Yes / No; because _____
2. Is $2^9 \cdot 3$ divisible by 3? Yes / No; because _____
3. Is $2^9 \cdot 3$ divisible by 8? Yes / No; because _____
4. Is $2^9 \cdot 3$ divisible by 9? Yes / No; because _____
5. Is it true that if a natural number is divisible by 4 and by 3, then it must be divisible by $4 \cdot 3 = 12$?

   Yes / No; because _____
6. Is it true that if a natural number is divisible by 4 and by 6, then it must be divisible by $4 \cdot 6 = 24$?

   Yes / No; because _____
7. A number A is even. Is it true that $3A$ must be divisible by 6?

   Yes / No; because _____
8. A number $5A$ is divisible by 3. Is it true that A must be divisible by 3?

   Yes / No; because _____
9. A number $15A$ is divisible by 6. Is it true that A must be divisible by 6?

   Yes / No; because _____
10. Is it true that the product of any three consecutive natural numbers is divisible by 6?

    Explain why.
11. Is it true that the product of any five consecutive natural numbers is divisible by 30?

    Explain why.

Earlier we had introduced the definition and notation

$$a \mid b \quad \Longleftrightarrow \quad b = ak$$

for some $k \in \mathbb{Z}$. The students were encouraged to work with this new notation and understand its meaning while we use it.

Next we established some basic properties of divisibility which will come handy as we proceed to problem-solving.

- Reflexivity
- Lack of symmetry
- Transitivity
- Adding and subtracting multiples: $a \mid b$ and $a \mid c$ implies $a \mid (b+c)$ and $a \mid (b-c)$
- Multiplying: $a \mid b$ and $a \mid c$ implies $a \mid bc$

2 Senior batch

This session continued the study of functions by moving from the algebraic ideas of composition and inverse functions introduced in the previous session to a geometric representation of functions through their graphs. The worksheet began with a short revision of inverses and composition, and then gradually introduced graphs of linear, quadratic, and cubic functions. The session concluded by connecting the graph of a function to the graph of its inverse.

The problems in the worksheet were as follows.

- Students were introduced to the graph of a function through
$$f(x) = x + 2.$$
They completed a table of values and plotted the corresponding points $(x, f(x))$. The purpose was to move from a numerical representation of a function to its geometric representation.

  Next we considered three collections of points and decided which could form part of the graph of a function. They were then asked to think about what would happen when a vertical line is drawn through a graph. This led naturally to the vertical line test as a graphical way of identifying a function.

- We considered the three linear functions
$$f(x) = x, \qquad g(x) = 2x, \qquad h(x) = 2x + 3$$
and sketched their graphs on the same coordinate plane. Students compared the three linear graphs and described what changes when the coefficient of x is changed and when a constant is added. This encouraged them to discover geometrically the roles played by the slope and vertical translation.

- Next we introduced to the quadratic function
$$f(x) = x^2.$$
They completed a table of values and sketched its graph. The activity encouraged observations about the shape and symmetry of a parabola. Starting from the graph of $y = x^2$, students were asked to sketch the graphs
$$y = x^2 + 2, \qquad y = x^2 - 3, \qquad y = 2x^2$$
without constructing new tables. They were encouraged to describe the geometric effect of each algebraic change, providing an introduction to transformations of graphs.

  Students also investigated the intersection of the graphs
$$y = x^2 \qquad \text{and} \qquad y = 2x + 3.$$
They found the intersection points algebraically and were asked to interpret the same calculation geometrically. The problem connected solving an equation with finding the intersection of two graphs.