

Krea – RAM – Maths Circle – Session 9

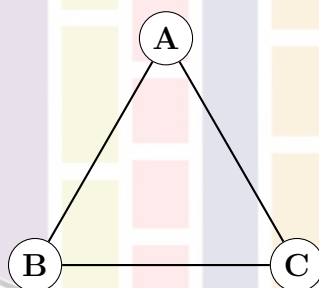
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1 Overview

We returned to path counting, this time on the triangle (K_3) and the four-point graph from Session 8 (K_4 minus the B – D edge). On the triangle we found a recursive formula for the number of paths and a closed-form expression.

2 The Triangle



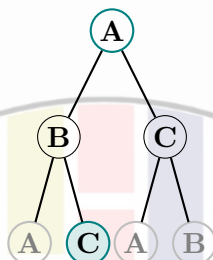
We counted the number of paths from A to C for lengths 1 through 7, along with the paths from A to A and from A to B .

n	$A \rightarrow A$	$A \rightarrow C$	$A \rightarrow B$
0	1	0	0
1	0	1	1
2	2	1	1
3	2	3	3
4	6	5	5
5	10	11	11
6	22	21	21
7	42	43	43

Because the triangle is symmetric, $A \rightarrow B$ and $A \rightarrow C$ always have the same count.

3 The Branching Tree

To see where the recursive formula comes from, think of each path as a branch in a tree. Starting at A , you branch to each neighbor at every step. Here is the tree for length-2 paths from A :



The leaves labeled C are the paths that arrive at C : there is 1, matching the table.

To reach C at step $n + 1$, at step n you must be at a neighbor of C . In the triangle, C 's neighbors are A and B . So:

$$(A \rightarrow C \text{ in } n + 1) = (A \rightarrow A \text{ in } n) + (A \rightarrow B \text{ in } n)$$

By symmetry $A \rightarrow B$ in n steps equals $A \rightarrow C$ in n steps. Call this s_n .

For $A \rightarrow A$ in n steps: at step $n - 1$ you must be at a neighbor of A , which is B or C . By symmetry both contribute s_{n-1} , so $A \rightarrow A$ in n steps $= 2s_{n-1}$.

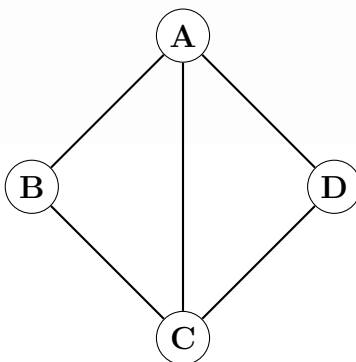
Substituting:

$$s_{n+1} = s_n + 2s_{n-1}$$

with $s_1 = 1$, $s_2 = 1$.

We also observed that s_n is always the nearest integer to $\frac{2^n}{3}$.

4 The Four-Point Graph



We applied the same branching approach to the four-point graph from Session 8. Here the tree grows faster because most vertices have three neighbors instead of two. The path counts from A to C :

Length	Paths from A to C
1	1
2	2
3	5
4	14
5	33
6	88

Unlike the triangle, the four-point graph is not fully symmetric (since B and D are not connected), so a single recurrence for $A \rightarrow C$ alone does not simplify as cleanly.

5 Challenge

Can you find a recursive formula for the path counts on the four-point graph? You may need to track more than one sequence at the same time.

