IMSc - RAM Math Circle - Chennai

Synopsis for September 6, 2026

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1 Tic-Tac-Toe exploration

Today we worked on developing a strategy for tic-tac-toe, a very popular game among kids.

The goal was to let students experience the progression

$$\text{play} \longrightarrow \text{notice patterns} \longrightarrow \text{formulate a strategy}$$
$$\longrightarrow \text{test the strategy} \longrightarrow \text{ask whether it is guaranteed.}$$

The important distinction at the end is between

"I have played many games and this seems to work"

and

"I can give an argument showing that this strategy cannot fail."

The exploration proceeded as follows:

- Students formed pairs and played several games to find out if there is a preferred/recommended starting position (edge/corner/center).
- After playing many games, the students formulated a strategy:
  - If I can win at the next turn, I should play to win.
  - If my opponent can win next turn, I should block.
- The students observed that they need to make a distinction between *losing* and *not winning*.
- The students more-or-less agreed that if both sides play perfectly, then neither player can win or lose, and the game ends in a draw.
- The discussion ended by asking the students to think about whether the following two statements mean the same thing or not:
  - *"I have never lost when I use this strategy."*
  - *"I can prove that this strategy means I will never lose."*

2 Modular arithmetic

After a short break, we picked up our discussion of modular arithmetic that we had started in the previous session. In particular,

- we reviewed the definition of congruency modulo n.
- we discussed what it means to say a *divides* b, when we are working in the set of integers.
- we reinforced both concepts through a number of examples.
- we discussed why the following two statements could be equivalent:

$$a \equiv b (mod\ n) \qquad \text{and} \qquad n \text{ divides } (b-a).$$

Last time we had introduced modular arithmetic as a way to sort integers into baskets. In this session we introduced the mathematical language used to express this idea and work with it.

Definition: Let a, b and n be integers. We say that *a is congruent to b modulo n* if n divides $(b-a)$, that is $(b-a)$ is a multiple of n.

We can quickly check how this applies with a few examples -

Examples:

1. $5 \equiv 3 (\text{mod } 2)$ since 2 divides $(5-3)$
2. $12 \equiv 7 (\text{mod } 5)$ since 5 divides $(12-7)$
3. $8 \equiv 15 (\text{mod } 7)$ since 7 divides $(15-8)$
4. $15 \equiv 27 (\text{mod } 3)$ since 3 divides $(27-15)$

The students enjoyed working on the following True/False exercise:

- $0 \mid 1$
- $1 \mid 1$
- $1 \mid 0$
- $0 \mid 0$
- $0 \mid n$, for any integer n
- $1 \mid n$, for any integer n
- $n \mid 0$, for any integer n
- $n \mid 1$, for any integer n
- $n \mid n$, for any integer n
