

This was the first session of the new academic year. We started with an exploration of walks along polygons.

Walking on a polygon

The aim of this activity was for students to discover that a walk visits every vertex exactly once precisely when the step size and the number of vertices are coprime.

- The pentagon with step size 2 produces the walk

$$0 \rightarrow 2 \rightarrow 4 \rightarrow 1 \rightarrow 3 \rightarrow 0.$$

Every vertex is visited exactly once, the walk returns after 5 steps, and the picture is a five-pointed star.

- For the hexagon,

1. Step size 2 produces

$$0 \rightarrow 2 \rightarrow 4 \rightarrow 0,$$

so only the even-numbered vertices are visited.

2. Step size 3 produces

$$0 \rightarrow 3 \rightarrow 0,$$

so only two vertices are visited.

3. Students explored many other possible step sizes and noticed that not every walk reaches every vertex.

- For the octagon, the completed table is

Step size	Visited every vertex?	Returned after how many steps?
1	Yes	8
2	No	4
3	Yes	8
4	No	2

Many students notice that 1 and 3 work while 2 and 4 do not. Resist introducing the greatest common divisor at this point.

- On the 12-gon, the behaviour is summarised below.

Step size	Behaviour
1	Visits every vertex
2	Visits only the even vertices
3	Visits four vertices
4	Visits three vertices
5	Visits every vertex
6	Visits two opposite vertices
7	Same picture as step size 5 (reverse direction)
8	Same picture as step size 4
9	Same picture as step size 3
10	Same picture as step size 2
11	Same picture as step size 1

- For different n -gons, the step sizes that give complete walks are:

Polygon	Successful step sizes
5	1, 2, 3, 4
6	1, 5
8	1, 3, 5, 7
10	1, 3, 7, 9
12	1, 5, 7, 11

- The expected observation is that every successful step size has no common factor with the number of vertices. After students expressed this idea in their own words, we introduced the term *coprime* (which some students knew already).
- Eventually students could formulate a rule equivalent to

A walk visits every vertex exactly once before returning to the start if and only if the step size and the number of vertices are coprime.