RAM Math Circle - Chennai

Session Report - December 8 2024

In the last session some concepts from graph theory were introduced. In this session we continue the work and discuss a few more ideas. [1]

1 Eulerian walks and circuits

Definition: An *Eulerian walk* is a given graph G is a sequence of edges of G such that every edge occurs exactly once in the sequence. The vertices may be repeated.

We call such a sequence an *Eulerian circuit* if the sequence starts and ends at the same vertex.

In the previous session it was observed that:

- If a graph has a closed Eulerian walk, then the degree of any vertex in the graph must be even.
- If a graph has an Eulerian walk that is not closed, then there are exactly 2 vertices that have odd degree, and all other vertices have even degree.

Children often see examples of graphs in the form of puzzles or other recreational activities and come across instructions such as 'retrace the graph without lifting your pencil and without repeating any edge'. We noted that this instruction is essentially asking the child to trace out an Eulerian walk in the graph.

We worked on various graphs trying to trace out an Eulerian path and/or Eulerian circuit in the graph.

2 Vertex colouring

Given a graph G, the vertex coloring problem is to colour the vertices in such a way that no two adjacent vertices have the same colour. Recall that two vertices are said to be adjacent if there is an edge between them.

The least number of colours used in vertex colouring is called the *chromatic number* of a graph.

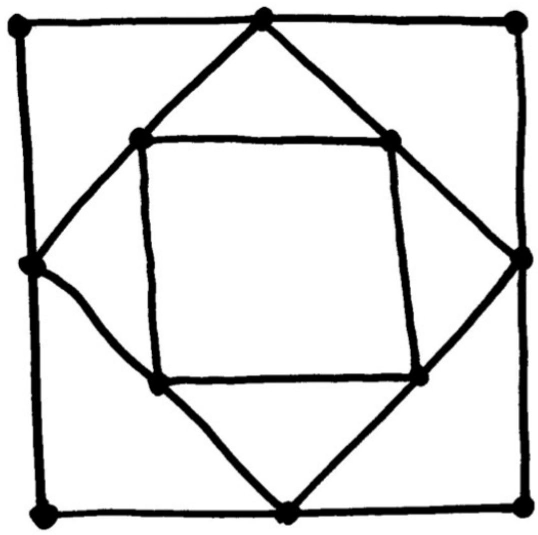
3 Map colouring

The map colouring problem is simlar to the vertex colouring problem, except that instead of vertices there are regions of a map to be coloured. Two regions are said to be adjacent if they share a boundary. The problem is to colour a given map so that no two adjacent regions have the same colour. We try to do this with as few colours as possible.

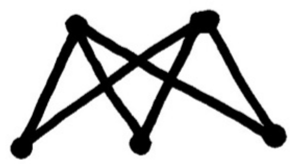
[1] The activities for this session were adapted from the work of Joel Hamkins `https://jdh.hamkins.org/math-for-eight-year-olds/`

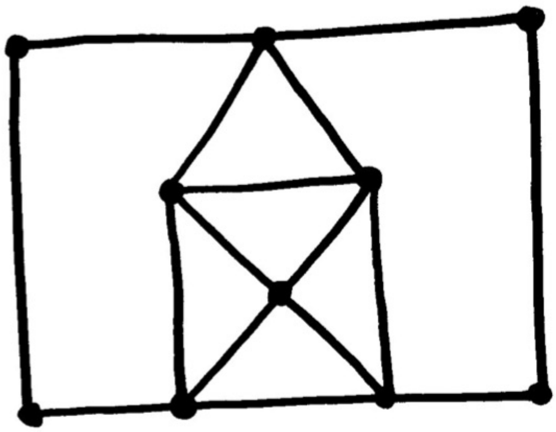
EULERIAN PATHS & CIRCUITS

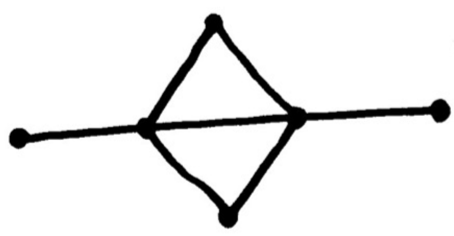
DRAW THESE SHAPES WITHOUT LIFTING YOUR PENCIL AND WITHOUT RETRACING ANY LINE.



A CIRCUIT STARTS AND ENDS IN THE SAME PLACE.

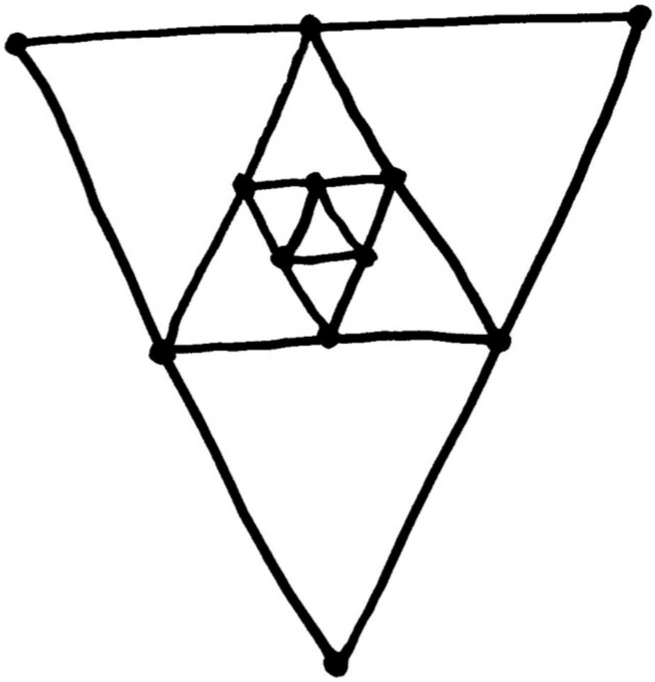


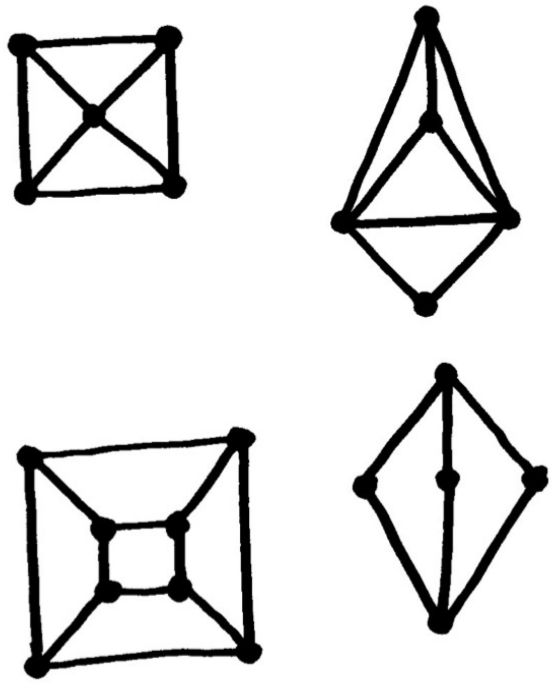




A PATH CAN START AND END IN DIFFERENT PLACES.

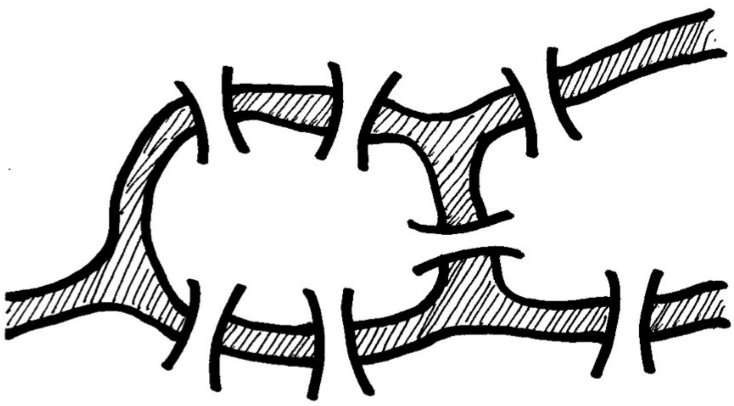
ONLY SOME OF THESE GRAPHS HAVE AN EULERIAN PATH OR CIRCUIT. CIRCLE THE IMPOSSIBLE GRAPHS.



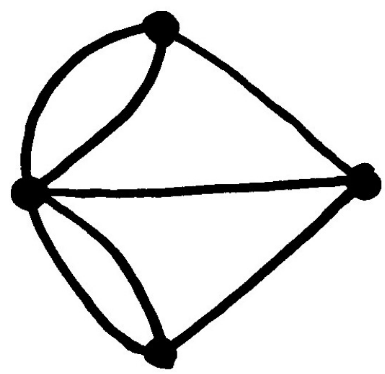


THE SEVEN BRIDGES OF

KÖNIGSBERG

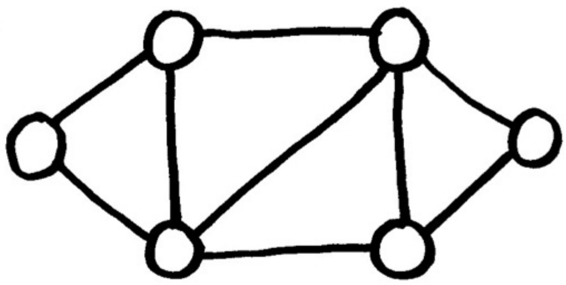


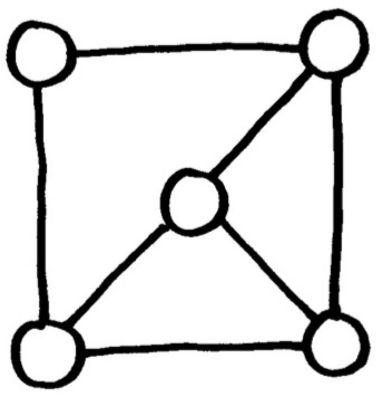
IS IT POSSIBLE TO TOUR THE CITY, CROSSING EACH BRIDGE EXACTLY ONCE?



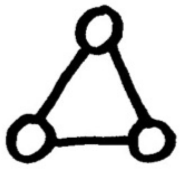
IS THERE AN EULERIAN PATH? EVERY NODE HAS ODD DEGREE.

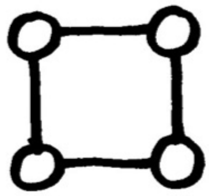
Vertex colouring

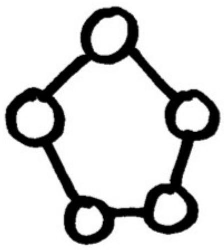


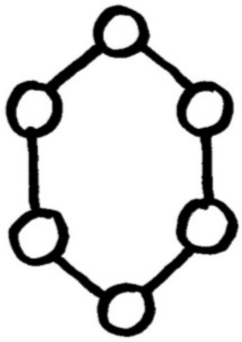


COLOR EACH VERTEX SO THAT CONNECTED VERTICES HAVE DIFFERENT COLORS.



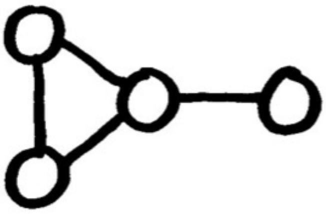


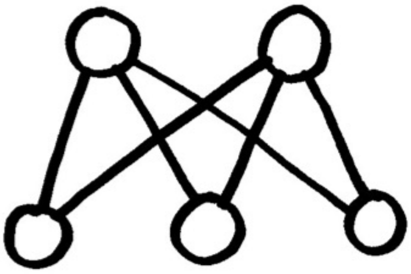


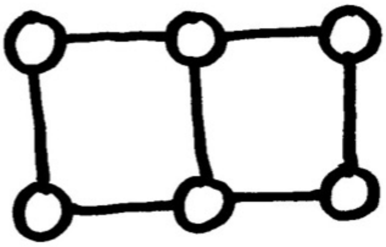


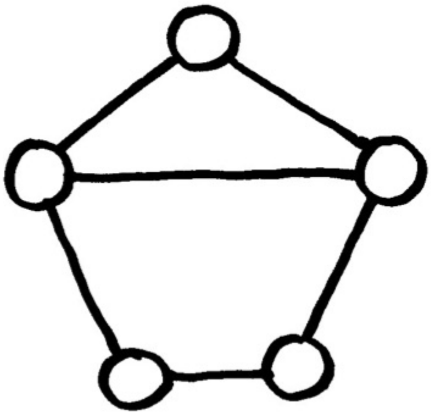
TRY TO USE THE FEWEST NUMBER OF COLORS — THIS IS THE CHROMATIC NUMBER.

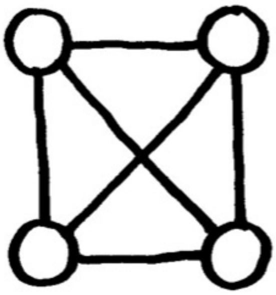
WHAT IS MY CHROMATIC NUMBER?

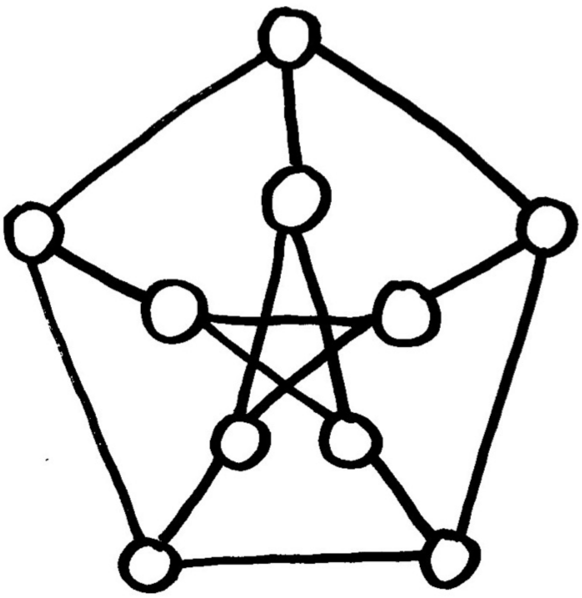


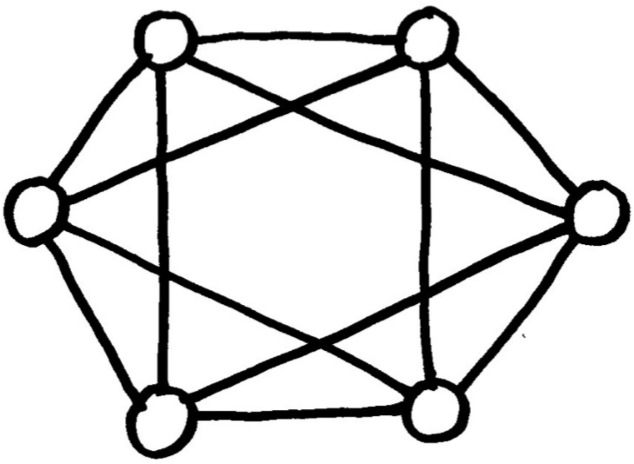










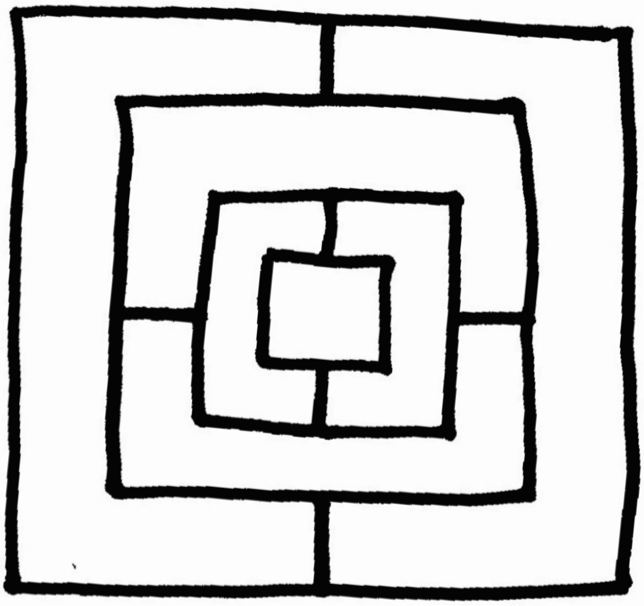


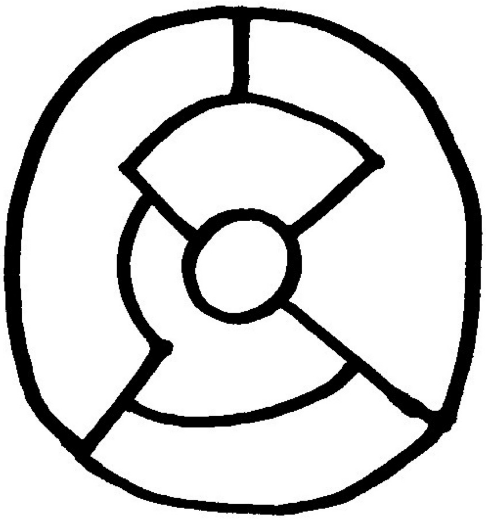
Create your own graph as a challenge for your partner; see who can colour it with fewer colours.

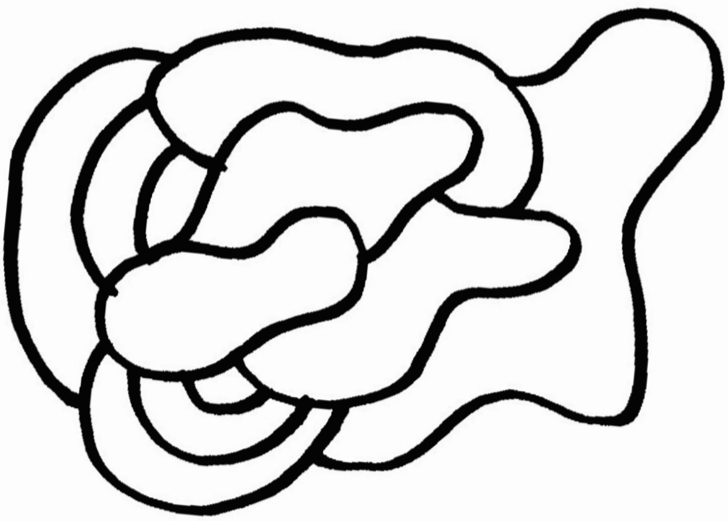
Coloured by ________ using ______ colours	Coloured by me using ______ colours

Map colouring

Colour the countries on this map so that adjacent countries have different colours.







Try to use the fewest possible colours.

Create your own map and colour it with the fewest possible colours.