
IIIT Delhi - RAM Maths Circle

Session 6

(Organized by the Department of Mathematics, IIIT Delhi)

IIIT Delhi

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Last session we saw that a 3×3 magic square is very rigid: the magic sum is forced, the centre entry is forced, and three numbers x, p, q describe every one of them. Today we move to 4×4 , where that rigidity collapses – and we spend the leftover freedom on building a magic square whose first row is your own date of birth.

Problem 1. Complete each square. First, determine its magic sum. Then explain why the entries you found are forced. In (a), the sixteen entries are $1, 2, \dots, 16$, each used exactly once; in (b), they are not; in (c), only the first row is given.

16			13
5	10		8
	6	7	12
	15	14	

(a)

8	-5		5
		3	0
1	-2		4
-4	7	6	

(b)

1	2	3	4

(c)

- (i) In the third square, what must the magic sum be?
- (ii) Find one completion of (c). Then find a second one that is not obtained from the first by any symmetry of the square.

Problem 2: (*Magic squares can be added.*)

Let M and N be 4×4 magic squares with magic sums S_M and S_N .

- (a) Prove that $M + N$ (added entry by entry) is magic, and find its magic sum. Prove that λM is magic for every number λ .
- (b) Call a square a *null square* if every row, every column and both diagonals sum to 0. Prove that $M + Z$ is magic with the same magic sum as M , for every null square Z .
- (c) Show that Session 05, Problem 3 (replacing x by $ux + v$) is a special case of (a).

Problem 3. Place the four symbols A, B, C, D in a 4×4 grid so that every row, every column, and both diagonals contain each symbol exactly once.

- (a) Find such an arrangement. Placing them so that only the rows and columns work is easy; the two diagonals are the real constraint.
- (b) Now substitute any four numbers for A, B, C, D . Prove that the result is magic, and find its magic sum.
- (c) Put your birthday in the top row:

$$A = \text{day}, \quad B = \text{month}, \quad C = \text{first two digits of the year}, \quad D = \text{last two digits}.$$

For example, 9 June 1954 gives

$$A = 9, \quad B = 6, \quad C = 19, \quad D = 54.$$

You now have a magic square with your birthday in it. What is unsatisfying about it?

Problem 4. We now remove the repetitions from Problem 3 without disturbing the birthday.

- (a) Find a null square whose first row is $0 \ 0 \ 0 \ 0$.
Why is that condition exactly the one we need?
- (b) In your symbol square, circle the four cells holding A ; then the four holding B , and so on.
What must a null square do to those four cells if the four copies of A are to become four different numbers?
- (c) Check that the square Z below is a null square with first row zero, and work out what each of A, B, C, D receives from it.
Then add Z to your symbol square and write the result in terms of A, B, C, D .

$$Z = \begin{array}{|c|c|c|c|} \hline 0 & 0 & 0 & 0 \\ \hline 1 & -1 & -3 & 3 \\ \hline -2 & 2 & 2 & -2 \\ \hline 1 & -1 & 1 & -1 \\ \hline \end{array}$$

- (d) Using the matrix Z , construct your birthday magic square.