IIIT Delhi - RAM Maths Circle
Session 12

(Organized by the Department of Mathematics, IIIT Delhi)

IIIT-Delhi December 7th, 2025

Problem 1. Is the number

$$|2^{3000} - 3^{2006}|$$

bigger or smaller than $\frac{1}{2}$

Problem 2. A six-digit number having 1 as its leftmost digit becomes three times bigger if we take this digit off and put it and the end of the number. What is this number?

Problem 3. Find all positive integers N and n such that N^2 differs from $n(N+n)$ by 1.

Problem 4. Let $S \subset \mathbb{N}$ be a set of positive integers whose product is 2021 times its sum. Let $S \subset \mathbb{N}$ be a set of positive integers whose product is 2021 times its sum.

(a) Given that S has five (or more) elements, what is the minimum possible value of its sum?

(b) Given that S has exactly four elements, what is the minimum possible value of its sum?

(c) Given that S has exactly three elements, what is the minimum possible value of its sum?

Problem 5. Naina, Payel, Rumi, Tina, Anjali, and Diksha all study in the same class. However, not all of them are friends. In fact, each of them does not get along well with exactly two others and is friends with the remaining people. All of them are invited to a birthday party and asked to be seated around a circular table.

(a) Will they all manage to sit comfortably, ensuring that everyone sits between two friends?

(b) Now, consider the following generalized situation: Suppose $2n$ people are attending a party, where each person does not get along well with at most $n-1$ others and has friendly relations with the remaining people. Is it possible for all of them to sit comfortably around a circular table so that each person sits between two people he/she is friends with?

End of problem sheet