RAM - Maths Circle

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Some problems taken from Russian math circle problem set.

1. Problems

1. Which number is greater: 31^{11} or 17^{14}?

2. Show that $2^{100} + 3^{100} < 4^{100}$

3. Prove that

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} \ldots - \frac{1}{99} + \frac{1}{100} > \frac{1}{5}$$

4. Which number is greater: 1234567×1234569 or 1234568^2?

5. How many ways are there to rearrange the letters in "ASUNDER" so that the vowels will be in alphabetical order, as well as consonants? Example: "DANERUS" is one way since A-E-U and D-N-R-S appear in order.

6. Find the remainder when 3^{102} is divided by 101.

7. Given that p, $p + 10$ and $p + 14$ are prime numbers, find p.

8. Given the pair of prime numbers p and $8p^2 + 1$, find p.

9. Prove that there are no natural numbers a and b such that $a^2 - 3b^2 = 8$.

10. In an 8×8 table one of the boxes is colored black and all the others are white. Prove that one cannot make all the boxes white by "recolouring" the rows and columns. "Recolouring" is the operation of changing the color of all the boxes in a row or column. Is that true for a 3×3 table also?