

RAM Math Circle - Chennai
Synopsis for September 27 2025

We began by recalling the definition of $n!$ and explored how many zeros appear at the end of $n!$. Students noticed that each trailing zero comes from a factor of $10 = 2 \times 5$. Since factorials contain more factors of 2 than 5, the count of trailing zeros equals the number of times 5 appears as a factor in the numbers $1, 2, \dots, n$.

$$\# \text{ of trailing zeros in } n! = \left\lfloor \frac{n}{5} \right\rfloor + \left\lfloor \frac{n}{25} \right\rfloor + \left\lfloor \frac{n}{125} \right\rfloor + \dots$$

(Continue until the quotient is 0.) Here,

- **Floor:** $\lfloor x \rfloor$ is the greatest integer $\leq x$.
- **Ceiling:** $\lceil x \rceil$ is the least integer $\geq x$.

Example:

- 5!: $\lfloor 5/5 \rfloor = 1$ zero.
- 17!: $\lfloor 17/5 \rfloor + \lfloor 17/25 \rfloor = 3 + 0 = 3$ zeros.
- 100!: $\lfloor 100/5 \rfloor + \lfloor 100/25 \rfloor = 20 + 4 = 24$ zeros.

Students understood that the problem reduces to counting factors of 5, including multiples like 25, 125, etc., which contribute additional 5s. Most could apply the formula independently; a few initially counted only $\lfloor n/5 \rfloor$ and forgot higher powers of 5, but corrected this with practice.

Then we recorded that for a prime p and positive integer n , the maximal power of p dividing $n!$ is

$$v_p(n!) = \sum_{k=1}^{\infty} \left\lfloor \frac{n}{p^k} \right\rfloor,$$

where the sum is finite in practice since $\lfloor n/p^k \rfloor = 0$ once $p^k > n$.

Next we recalled the following definitions from earlier sessions.

- **Binomial coefficient:** $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for $0 \leq r \leq n$.
- **Pascal's rule:** $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$, with edges $\binom{n}{0} = \binom{n}{n} = 1$ and symmetry $\binom{n}{r} = \binom{n}{n-r}$.

After recalling these definition we discussed the following question. **Question:** What is the maximal power of a prime p dividing a binomial coefficient $\binom{n}{r}$?

Solution: Using the earlier result for $v_p(n!)$,

$$v_p\left(\binom{n}{r}\right) = v_p(n!) - v_p(r!) - v_p((n-r)!) = \sum_{k \geq 1} \left(\left\lfloor \frac{n}{p^k} \right\rfloor - \left\lfloor \frac{r}{p^k} \right\rfloor - \left\lfloor \frac{n-r}{p^k} \right\rfloor \right),$$

and the sum stops when $p^k > n$.

Pascal Triangle Modulo 2, 3, 5 (20 rows)

Students were given 20 rows of Pascal's triangle and asked to reduce entries modulo 2, 3, and 5.

What to do ?(instructions)

1. Compute each entry with Pascal's rule and then reduce the value mod p .
2. Shade or mark entries congruent to 0 (multiples of p), optionally color by residue class.
3. Compare patterns across $p = 2, 3, 5$.

Observations made by the students:

- Sierpiński triangle appears with many triangular translational symmetries.

- **Modulo 2:**

- The row with index $2^n - 1$ is entirely 1's (no zeros).

- **Modulo 3:**

- The row with index $3^n - 1$ contains only 1's and 2's (no zeros).
- The row with index $3^n + 1$ has the form

$$1, \underbrace{0, \dots, 0}_{\text{a block of zeros}}, 1,$$

i.e., it begins with two 1's, then a stretch of zeros, and ends with two 1's.

- **Modulo 5:**

- The row with index $5^n - 1$ contains only 1's and 4's (no zeros).