Synopsis for February 15 2026

Junior Batch

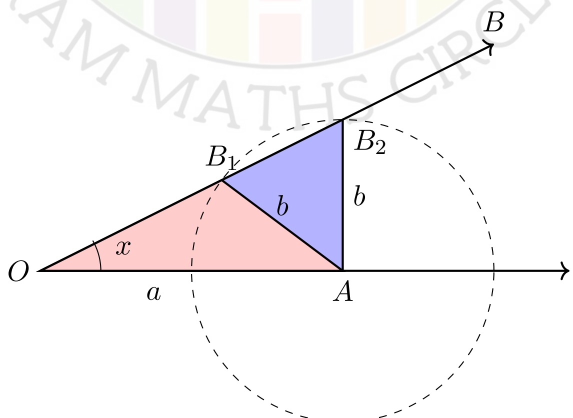
We began this session by recalling definitions and the results from the previous two sessions (January 24th) and (February 08th). In this session we explored Congruency and similarity of triangles and other geometric shapes.

Two geometric figures are **congruent** if they have the same shape and the same size. Two geometric figures are **similar** if they have the same shape but may differ in size. In other words, corresponding angles are equal and corresponding sides are in the same ratio.

Criterion	**Given Condition**	**Congruent?**
SSS	Three corresponding sides are equal	Yes
SAS	Two sides and the included angle are equal	Yes
ASA	Two angles and the included side are equal	Yes
AAS / SAA	Two angles and a corresponding side are equal	Yes
RHS	Right angle, hypotenuse, and one side are equal	Yes
SSA / ASS	Two sides and a non-included angle	No
AAA	Three angles are equal (similarity only)	No

Why SSA (ASS) is not a Congruence Criterion

Let us fix an angle $\angle AOB = x$. Let the side $OA = a$ be fixed. Now say the third side $AB = b$. All points which are at a distance b from the point A lie on a circle with centre A and radius b. The ray OB intersects this circle at two distinct points, say B_1 and B_2. Thus, two triangles $\triangle OAB_1$ and $\triangle OAB_2$ are formed.



These two triangles have two equal sides and the same non-included angle, but they are not congruent, since their shapes are different.

Hence, **SSA (or ASS) is not a congruence criterion**.

Similarity criteria:

- **AA**: Two corresponding angles are equal

- **SAS (Similarity)**: Included angle equal and surrounding sides in the same ratio
- **SSS (Similarity)**: Corresponding sides are in the same ratio

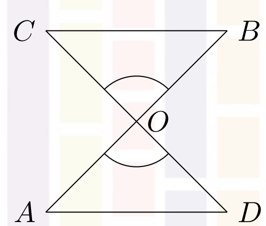
Any two	**Similar?**	**Reason**
Square	Yes	Angles equal, sides in proportion
Circle	Yes	Shape depends only on radius
Rectangle	No	Length : breadth not fixed
Rhombus	No	Angles not fixed
Parallelogram	No	Angles and side ratios vary

Read the table as follows: Any two square are similar because their corresponding angles are equal and their corresponding sides are propotional.

In the following figure, $OA = OB$ and $OD = OC$. and it is given that AB and CD are straight lines. Show that

(i) $\triangle AOD \cong \triangle BOC$ and

(ii) $AD \parallel BC$.



Solution:
(i) In $\triangle AOD$ and $\triangle BOC$, we are given that

$$OA = OB \qquad OD = OC$$

Also, $\angle AOD$ and $\angle BOC$ are vertically opposite angles. Hence,

$$\angle AOD = \angle BOC.$$

Therefore,

$$\triangle AOD \cong \triangle BOC \quad \text{(by SAS congruence rule).}$$

(ii) In congruent triangles AOD and BOC, corresponding parts are equal. So,

$$\angle OAD = \angle OBC.$$

These form a pair of alternate interior angles for the lines AD and BC.

$$\therefore AD \parallel BC.$$

Senior Batch

We continued our exploration of synthetic division. The following exercises were provided in a worksheet:

1. Divide $x^3 - 22x + 13$ by $x + 3$.
2. Divide $8x^4 - 2x^3 + 7x^2 - 5x + 1/3$ by $2x + 5$.
3. Solve the equation $2x^3 - 3x^3 - 11x + 6 = 0$ given that -2 is a zero of $f(x) = 2x^3 - 3x^3 - 11x + 6$.
4. Write down the equations and procedure for synthetic division when we divide $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ by $ax^2 + bx + c$.

   Special case: Assume that $a = 1$ for convenience

   General case:
5. Divide $5x^5 + 13x^4 + 2x^2 - 7x - 10$ by $x^2 + 2x - 3$.
6. **Generalised synthetic division:** Write down the equations and procedure for synthetic division when we divide $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ by $b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0$, where $a_n \neq 0$, $b_m \neq 0$ and $m \leq n$.

   Special case: Assume that $b_m = 1$ for convenience

   General case:
7. Divide $5x^5 + 13x^4 + 2x^2 - 7x - 10$ by $2x^3 - 3x^3 - 11x + 6$.

We noted that by applying the division algorithm to polynomials $a(x)$ and $c(x)$, with degree $a(x) = n$ and degree $c(x) = 2$, we get that the quotient $b(x)$ must have degree $(n-2)$ and the remainder must have degree less than or equal to 1. So we may write $a(x) = c(x)b(x) + r(x)$ as:

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = (c_2 x^2 + c_1 x + c_0)(b_{n-2}x^{n-2} + b_{n-3}x^{n-3} + \cdots + b_1 x + b_0) + (r_1 x + r_0).$$

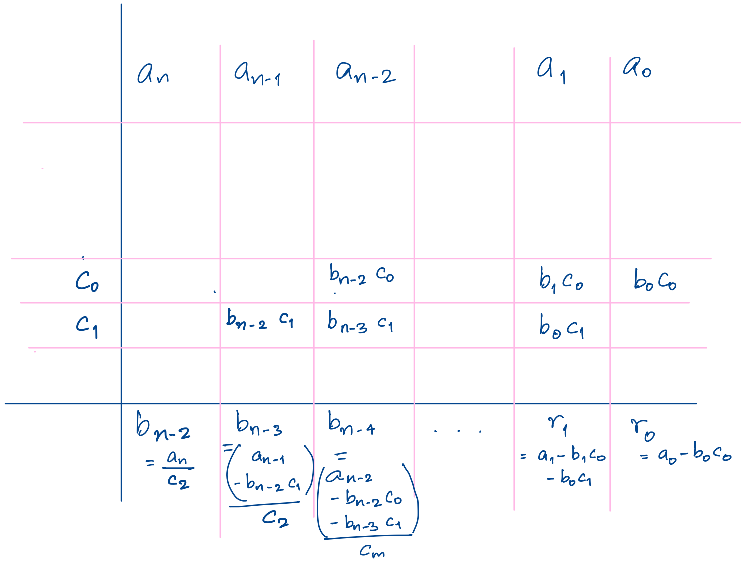
By expanding out the right hand side and equating coefficients of every x^i term, we get equations:

$$\begin{aligned}
c_2 b_{n-2} &= a_n \\
c_2 b_{n-3} + c_1 b_{n-2} &= a_{n-1} \\
&\vdots \\
c_2 b_0 + c_1 b_1 + c_0 b_2 &= a_2 \\
c_1 b_0 + c_0 b_1 + r_1 &= a_1 \\
c_0 b_0 + r_0 &= a_0
\end{aligned}$$

If we compute the values of b_i's and r_i's from these equations we get:

$$\begin{aligned}
b_{n-2} &= \frac{a_n}{c_2} \\
b_{n-3} &= \frac{a_{n-1} - c_1 b_{n-2}}{c_2} \\
&\vdots \\
b_0 &= \frac{a_2 - c_1 b_1 - c_2 b_0}{c_2} \\
r_1 &= a_1 - c_0 b_1 - c_1 b_0 \\
r_0 &= a_0 - c_0 b_0
\end{aligned}$$

This gives us the following procedure for synthetic division when the divisor has degree 2:



The same idea can now be extended to a divisor of degree m, where $m < n$. SUppose the polynomial $a(x) = a_nx^n + a_(n-1)x^{n-1} + \cdots + a_1x + a_0$ is to be divided by $c(x) = c_mx^m + c_{m-1}x^{m-1} + \ldots c_1x + c_0$. By the division algorithm we know that

$$a(x) = c(x)b(x) + r(x),$$

where the quotient $b(x)$ is a polynomial of the form $b(x) = b_{n-m}x^{n-m} + b_{n-m-1}x^{n-m-1} = \cdots + b_1x + b_0$ and the remainder $r(x)$ has degree $< m$, so $r(x) = r_{m-1}x^{m-1} + \cdots + r_1x + r_0$. The equations for synthetic division in this case will be as follows:

$$\begin{aligned}
c_mb_{n-m} &= a_n \\
c_mb_{n-m-1} + c_{m-1}b_{n-m} &= a_{n-1} \\
&\vdots \\
c_mb_0 + c_{m-1}b_1 + \ldots c_1b_{m-1} + c_0b_m &= a_m \\
r_{m-1} + c_{m-1}b_0 + \cdots + c_0b_{m-1} &= a_{m-1} \\
&\vdots \\
r_1 + c_1b_0 + c_0b_1 &= a_1 \\
r_0 + c_0b_0 &= a_0
\end{aligned}$$

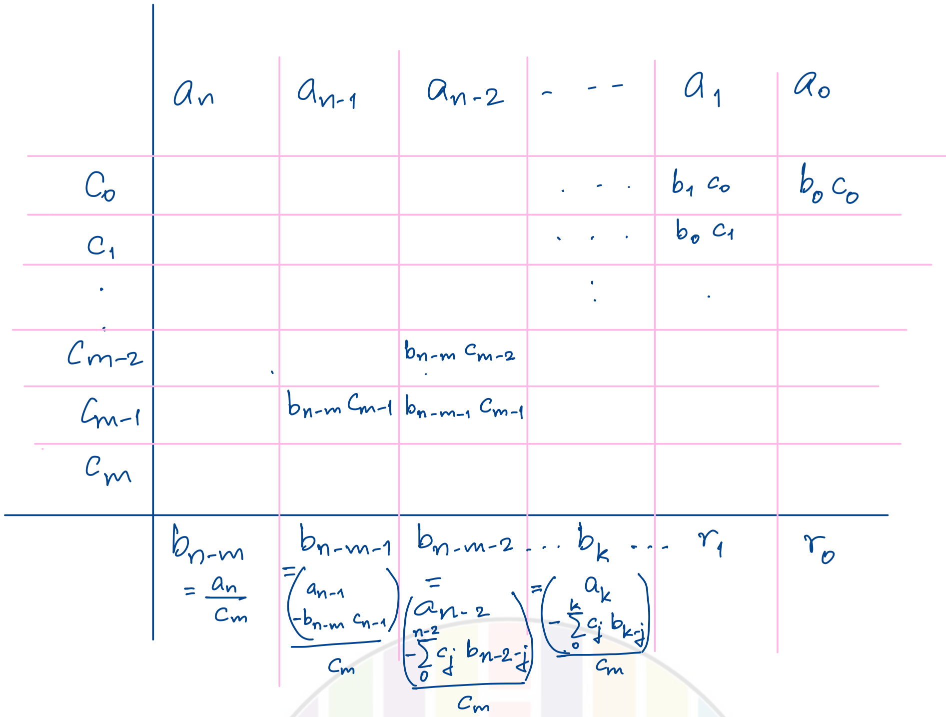
The expressions for b_i's and r_i's can be obtained in general as follows:

$$b_{k-m} = \frac{a_k - \sum_{j=0}^{k} c_jb_{k-j}}{c_m} \qquad \text{for } k = n, n-1, \ldots, m$$

and

$$r_k = a_k - \sum_{j=0}^{k} c_jb_{m-j} \qquad \text{for } k = m-1, m-2, \ldots, 0$$

The procedure for synthetic division takes the form:



When we began this exercise some students thought that long division was easier than this procedure. But by the end of the session, when the hard work was all done, most of them saw that the procedure can be stated quite easily and indeed it reduced the amount of work required in long division.

Another point to note is that since the degree of $a(x)$ is fixed, when the degree of the divisor increases, the degree of the quotient decreases. In terms of the above diagram, as the length of the table increases (because there are more number of coefficients in the divisor), we compute fewer terms in the last row!