Krea - RAM Maths Circle - Session 4

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1 Overview

Building on game theory, we got into voting theory through different voting methods and scenarios.

2 Fair Voting System

1. **Majority Criterion:** If more than half the voters rank someone first, they should win.
2. **Pareto Criterion:** If every voter prefers X over Y, then Y should not win.
3. **Independence of Irrelevant Alternatives (IIA):** Removing a losing candidate shouldn't change the winner.
4. **Monotonicity:** If the winner gains more support, they should still win.
5. **Non-Dictatorship:** One person shouldn't decide everything.

3 Voting Methods

1. **Plurality**
   - People vote for their top choice of candidate.
   - The candidate with the most votes wins.
   - What if the candidate who was ranked the best by most people, say n, was ranked the worst by more than n people? Is it fair?
2. **Instant Runoff (IRV)**
   - Eliminate the candidate with the fewest first place votes.
   - Transfer their votes to each voter's next choice.
   - It is used in Australian Elections, New York City Electoral Races.

3. **Borda Count**
    - Give points (weights) to each ranking. The first person gets n-1 points, second person gets n-2 and so on till the second-last person gets 1 point and the last person gets 0 points.
    - Multiply these points with the corresponding total number of votes. Person with the highest value wins the elections.
    - This method rewards broad support, not just first-place votes. A candidate can win even if they are not the first place winners.
4. **Condorcet**
    - Compare every pair of candidates head-to-head.
    - If a candidate beats everyone, they win.

4 Which type of election is the best?

- **Plurality:** Fails because of the Spoiler Effect (violates IIA criterion); if a person drops out of the election, then the result of the whole election can change. (Happened in Florida Elections 2000)
- **Borda Count:** Fails because it violates the majority criterion of fair voting systems.
- **Instant Runoff:** Fails because it violates the monotonicity criterion of fair voting systems; gaining support can make you lose.
- **Condorcet:** Fails because even when every voter has perfectly logical preferences, the group's preferences can go in a circle (Condorcet Paradox).
- Every method fails a criterion that we initially defined.
- **No ranked voting system with three or more candidates can satisfy all of the basic fairness criteria at the same time. It is mathematically impossible to find one.**