IIIT Delhi - RAM Maths Circle
Session 6

(Organized by the Department of Mathematics, IIIT Delhi)

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§1. Counting Principle

Combinatorics, a branch of mathematics that is devoted to the art of counting things, is an enjoyable topic that has many applications in mathematics and computer science. At the same time, working on a good combinatorics problem is as close to puzzle solving as math can get.

For the next several lessons, we will be studying combinatorics. While we will learn several useful formulas, our primary goal is not to memorize them, but to develop an understanding of where these formulas come from. We also will be solving problems that promote understanding of combinatorics principles, not the direct application of the formulas.

Some warm-up problems:

1. How many ways are there to choose ten people from a group of 30 men and 30 women, if the group must include at least one woman?

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2. Suppose we have two identical dice with 8 faces with numbers from 1 to 8. If these two dice are rolled, how many different outcomes are there? Note that $(1, 2)$ and $(2, 1)$ will be considered as the same output as the dice are identical. Can you generalize this? That is, what if n identical dice are rolled, then how many different outcomes are there?

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3. How many positive integers not exceeding 2000 are divisible by 7 or 11?

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4. Find the number of two digit even numbers with both distinct digits. Can you think of three digit even numbers and hence n digit even numbers?

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5. On a rainy day, five gentlemen M_1, M_2, M_3, M_4, M_5 attend a party after leaving their umbrellas in a checkroom. After the party is over, the umbrellas get mixed up and are

returned to the gentlemen in such a manner that none receives his own umbrella. In how many ways can this be happen?

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6. There are 10 students in a room. In how many ways can we choose four of them and give them some present?

Some thinking problems:

1. (**The Magical Binomial**) By the factorial of any nonnegative number n,

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 3 \cdot 2 \cdot 1.$$

For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$. We also set $0! = 1$.

The number of different ways to pick k items from a set of n elements is denoted by $\binom{n}{k}$, called n *choose* k. It is defined as

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}.$$

Find

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}.$$

First, try to take some specific n and then identify the pattern and make a guess of the sum.

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2. (**Counting Subsets**) A set A is said to be a subset of B (written as $A \subseteq B$) if for any $a \in A$, we have $a \in B$. That is, all the elements of A are contained in B.

For example, if $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$, then $A \subseteq B$. However, if $A = \{1, 2, 3\}$ and $B = \{1, 2, 5, a, b\}$, then A is not a subset of B, since $3 \in A$ but $3 \notin B$.

The *empty set*, denoted by $\varnothing$, is a set with no elements. By convention, $\varnothing$ is a subset of every set.

Let A be a set with 4 elements. How many subsets of A are there? Can you generalize this to a set containing n elements?

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3. Find the number of ordered quadruples (x_1, x_2, x_3, x_4) of positive odd integers such that $x_1 + x_2 + x_3 + x_4 = 98$.

Homework Problems:

1. How many positive integers not exceeding 2025 are multiples of 3 or 4 but not 5?
2. Twenty five boys and twenty five girls sit around a table. Prove that it is always possible to find a person both of whose neighbors are girls.
3. Determine the number of ways to choose five numbers from the first eighteen positive integers such that any two chosen numbers differ by at least 2.
4. A school kid has eight pairs of shoes, each with a sock and a shoe. For each pair, the sock must be worn first, then the shoe. In how many different orders can the child put them all on, if, for every pair, the sock must come before the shoe?
5. A teacher gives students a task: "Take any fraction between 0 and 1 and write it in its simplest form. Then multiply its numerator and denominator. For how many such fractions will the product be equal to 20!".
6. Imagine the numbers 1 through 18 are 18 seats in a row. You want to pick 5 seats so that no two picked seats are right next to each other — there must be at least one empty seat between any two picks. How many different sets of 5 numbers (or choices of 5 seats) can you make?

Students should keep in mind that even if a problem seems difficult at first, it is always worthwhile to return to it later with fresh ideas. Avoid the temptation to flip straight to the solutions! And remember, some problems may also have alternative solutions that do not rely on the usual counting or permutation principle.