

Krea - RAM Maths Circle - Session 7

Hurudaya Narasimhan

20/9/2026

1 Overview

Representation of some basic aspects of number theory in geometry such as the Cartesian plane, Pythagoras theorem and circles.

2 Numbers and their Representations

- What are negative numbers?
- What is an axis on a Cartesian Plane? How can we interpret it using basic direction?
- We have $x^2 + y^2 = k$ where x, y are variables (values that can change) and k is a constant value. Can we represent this equation as a shape for different values of x and y ? $\rightarrow$ Yes. If we bring this equation onto a Cartesian Plane, we get a circle.
- The circle passes through some integer points, and the rest are non-integer points. The points where both x and y are integers are called lattice points.
- The square of these special coordinates is represented k as a sum of two squares in all ways. k can also be represented as a sum of squares of negative numbers, because we saw that squares of negative numbers always gave a positive number.
- For circles that do not pass through any lattice points, the value of k in their equation cannot be expressed as a sum of two squares.

3 Pythagoras Theorem

- What is the Pythagoras Theorem? A perspective based on the exploration of the sum of 2 squares done in the previous class.
- Why is the Pythagoras Theorem correct? A proof using the area of 4 triangles and a square.
- Can we see the right angle triangles in the circles that were drawn on the Cartesian Plane? Yes. Pick a lattice point, draw a vertical line from it towards the x-axis, draw a line from the lattice points towards the origin.

4 Brahmagupta-Fibonacci Identity

- If we have two numbers, a and b , which can be represented as the sum of two squares, can we always represent $a \times b$ as a sum of two squares?
- Another way of asking the same question: If we have two circles, $x^2 + y^2 = m$ and $x^2 + y^2 = n$, each having lattice points, then must the circle $x^2 + y^2 = mn$ also have lattice points?
- The identity: $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$
- Proved using algebra.
- Another way of representation: $(a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 + (ad - bc)^2$

