

Krea - RAM - Maths Circle - Session 9

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14/12/2025

1 Overview

An introduction to the concept of infinity through the famous Hilbert's Hotel thought experiment, exploring how infinite sets behave differently from finite sets.

2 Hilbert's Grand Hotel

Imagine a hotel with infinitely many rooms, numbered $1, 2, 3, 4, \dots$ and every room is occupied.

1. **A new guest arrives. Can you accommodate them?**

- In a finite hotel with all rooms full, this would be impossible
- Can you find a way to make room for one more guest?
- What instructions would you give to the current guests?

2. **A bus with infinitely many new guests arrives. Can you accommodate all of them?**

- The bus has passengers numbered $1, 2, 3, 4, \dots$
- How would you assign rooms to everyone (old and new guests)?
- Write a formula: Guest n from the bus goes to room _____

3. **Infinitely many buses arrive, each with infinitely many passengers!**

- Bus k has passengers numbered $1, 2, 3, 4, \dots$
- Can you still fit everyone in?
- Hint: Think about how we listed the rationals, or consider prime numbers

3 Discussion and Reflection



Students worked through each scenario, discovering that:

- Moving each guest from room n to room $n + 1$ accommodates one new guest
- Moving each guest from room n to room $2n$ frees all odd-numbered rooms for an infinite bus
- Various methods involving primes and exponentiation were discussed.

4 Key Questions for Next Session

1. What does “same size” mean for infinite sets?

- The hotel always has “the same number” of guests after each rearrangement
- But we added infinitely many new guests!
- How can $\infty + 1 = \infty$ and $\infty + \infty = \infty$?

2. Are all infinities the same size?

- We could fit the natural numbers $\mathbb{N} = \{1, 2, 3, \dots\}$
- Could we fit all real numbers \mathbb{R} (including decimals)?
- Students are tasked with thinking about whether there might be “bigger” infinities

5 Takeaway

The key insight from Hilbert’s Hotel is that infinite sets can be put into one-to-one correspondence with proper subsets of themselves (something that intuitively feels wrong at first for people used to finite sets). This will lead us to a rigorous way of comparing the “sizes” of infinite sets in the next session.