RAM Math Circle - Chennai

Synopsis for July 13 2025

1 Similarity of triangles and an application to camera geometry

After a quick discussion recalling similarity tests for triangles, we discussed an application of similarity to an application that ubiquitous in camera geometry: the calculation of the magnification factor of a camera.

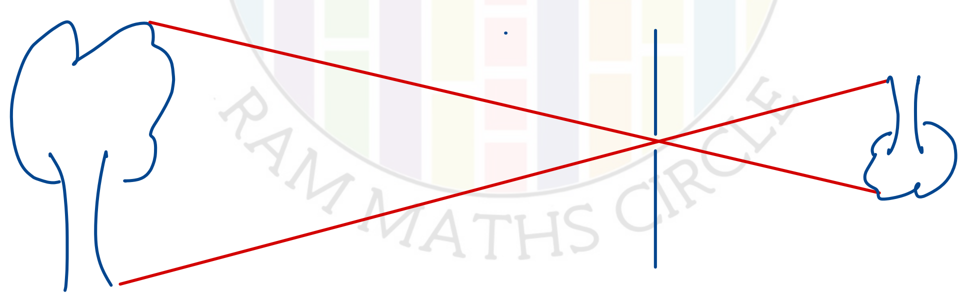
The use of cameras is everywhere, from mobile cameras in our hands to security cameras on streets and in stores to cameras mounted on drones for monitoring and delivery tasks. The following simple calculation, based on school geometry, lies at the heart of computations that help computer scientists and engineers use camera systems to make sense of the real world scenes that are recorded in pictures and videos.

For simplicity, we consider the pinhole camera model. A **pinhole camera** is the simplest way to think about how a real camera works. It has:

- A small hole (the "pinhole") that lets light in,
- A flat screen or film behind it to capture the image,
- No lens, just straight-line light rays!

How It Works

Light from a point on an object travels in a straight line through the hole and hits the screen. Because the light rays cross at the pinhole, the image appears **upside down**!



What It Shows

- Things that are **closer** to the hole appear **bigger** on the screen.
- Things that are **far away** look **smaller**.
- The image keeps the same shape, just scaled down and flipped.

Fun Fact!

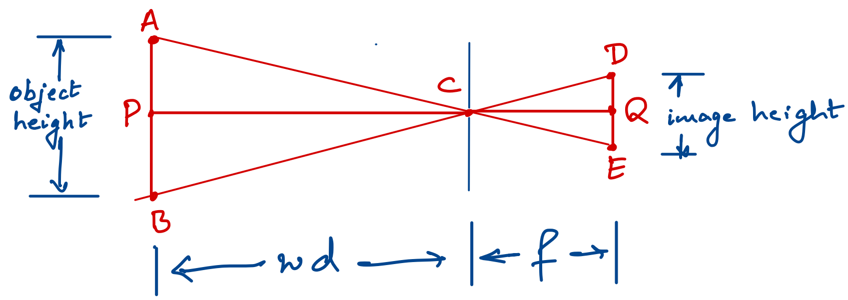
This idea is the basis for real cameras. In real life, cameras use lenses instead of pinholes — but the idea is the same: **light from the world gets projected onto a flat image**.

A connection with our history

Here's an article from *The Times of India* that describes the use of the pinhole effect in the Virupaksha temple in Hampi, Karnataka: "600 year old pinhole camera in Indian temple: How ancient architects built it centuries before modern optics".

Back to similar triangles

Notice that the pinhole camera setup as shown in Figure 1 can be stripped down to the following simple figure revealing the underlying geometric setting:



You may notice that we have a pair of similar triangles here.

$$\Delta ABC \sim \Delta EDC.$$

This gives us equality of ratios

$$\frac{AB}{ED} = \frac{AC}{EC} = \frac{BC}{DC} = \frac{CP}{CQ},$$

where CP and CQ are the altitudes constructed from C in the respective triangles. The first and the last ratios are important for us.

AB	$=$	object length (or height)
ED	$=$	image length (or height)
CP	$=$	perpendicular distance from pinhole (or lens) to object (also called the **focal length of the camera** (f)
CQ	$=$	perpendicular distance from pinhole or lens to image (or image screen or the camera sensor) (also called the **working distance** (**wd**))

The magnification factor of a camera is defined as the ratio of image height to object height. In other words, if

$$m = \frac{\textit{image height}}{\textit{object height}}$$

then the height of the object in real life is m times the height of the object in the image.

From our equations we get

$$m = \frac{\textit{image height}}{\textit{object height}} = \frac{ED}{AB} = \frac{CQ}{CP} = \frac{\textit{working distance}}{\textit{focal length}}$$

Thus, the magnification factor of a camera system can be calculated using either the image and object heights or using the working distance and the focal length.

So the next time you use a camera, take a moment to appreciate these simple ratios that come about in the image capturing process and make many further computations possible!

2 Problems on divisibility and congruences

In order to recall and refresh the concepts studied before the break, we worked on the following problems from the book *Mathematical Circles*.

1. Reduce 6^{100} modulo 7.
2. Prove that $30^{99} + 61^{100}$ is divisible by 31.
3. Prove that
   (a) $43^{101} + 23^{101}$ is divisible by 66.
   (b) $a^n + b^n$ is divisible by $(a + b)$ when n is odd.
4. Prove that $1^n + 2^n + \cdots + (n-1)^n$ is divisible by n for odd n.
5. For $n \in \mathbb{N}$, prove that if the last digit of n^2 is 6, hen the second-last digit of n^2 is odd.
6. The second-last digit of the square of a natural number is off. Prove that the last digit of the square is 6.