IIIT Delhi - RAM Maths Circle
Session 7

(Organized by the Department of Mathematics, IIIT Delhi)

October 26, 2025

We started off the session by handing the students a problem set as the students had an idea of the concepts used in the problems from previous sessions. The problem set focused mostly on combinatorics.

We continued working on counting problems based on case work and the addition & multiplication principles:

> **Addition principle:** If there are m different elements in set A and n different elements in set B, then the total number of ways of choosing 1 element from either set A or set B is $m + n$.
>
> **Multiplication principle:** If there are m different elements in set A and n different elements in set B, then the total number of ways of choosing an element from set A and an element from set B is $m \times n$.

Problems

1. The Hermetian alphabet consists of only three letters: A, B, and C. A word in this language is an arbitrary sequence of no more than four letters. How many words does the Hermetian language contain?

2. How many six-digit numbers have at least one even digit?

3. Which number is greater: 50^{99} or $99!$?

4. In a certain year there were exactly four Fridays and exactly four Mondays in January. On what day of the week did the 20th of January fall that year?

5. How many six-digit numbers have all their digits of equal parity (all odd or all even)?

6. How many nine-digit numbers have an even sum of their digits?

7. A "necklace" is a circular string with several beads on it. It is allowed to rotate a necklace but not to turn it over. How many different necklaces can be made using 13 different beads?

Most students were able to solve problems 1, 2, 4, 5 and 7 using case work and the addition & multiplication principles.

Near the ending hour of the session, students came up and showcased their solutions with the help of the faculty. The following problems were discussed:

Problem 3

They both have 99 factors and the middle one is 50 for both.

Here we notice that the pairs around them is $50 \times 50 = 2500$, and $49 \times 51 = 2499$. Similarly, the next pair is $50 \times 50 = 2500$, and $48 \times 52 = 2496$.

So for each pair, 50×50 "wins" by more and more, until the last pair, $50 \times 50 = 2500$, and $1 \times 99 = 99$.

So, 50^{99} wins out over 99! by quite a wide margin and is thus the greater number.

Problem 6

For this problem, we discussed three solutions:

A nine-digit number has digits $d_1d_2d_3d_4d_5d_6d_7d_8d_9$. The first digit, d_1, can be any digit from 1 to 9 (9 choices). The remaining eight digits, d_2 to d_9, can be any digit from 0 to 9 (10 choices each). For a sum of digits to be even, the number of odd digits must be even.

i) **Case-work:** Using case work, we can divide the problem into 5 cases to get:

- case 1 – 9 even digits
- case 2 – 7 even digits, 2 odd digits
- case 3 – 5 even digits, 4 odd digits
- case 4 – 3 even digits, 6 odd digits
- case 5 – 1 even digit, 8 odd digits

Then calculate the number of numbers in each case and sum them to get the answer; this is straightforward but lengthy. Students are encouraged to try this approach.

ii) In a 9-digit number, the first digit will have 9 choices. For the rest of the 8 digits, we can fix 1 digit to have only odd digits or only even digits (5 choices for each), and for the other 7 digits we have 10 choices for each.

Using this, we get that:

$$\text{Number of 9-digit numbers having an even sum of digits} = 9 \times 5 \times 10^7 = 450{,}000{,}000.$$

iii) There are 900,000,000 nine-digit numbers and exactly half of them have an even sum of digits (because every number can be paired with another of the opposite parity by changing the last digit). Hence exactly

$$\frac{900{,}000{,}000}{2} = 450{,}000{,}000$$

nine-digit numbers have an even sum of digits.