RAM Math Circle - Chennai

Session report for July 20 2025

1 Synopsis

This Sunday's session started off with a problem in school geometry based on similarity of triangles. The students required some time to set it up, and with a hint about the construction most of them could solve the part about the interior angle.

The geometry problem got the kids a bit tired, so we skipped problems 2 and 3 and moved to problem 4. This one really perked everyone up. We had some real chocolates to liven up the mood too. Some intense group discussions followed, with the TAs also taking part animatedly in the discussions. Students formed pairs competing with each other using different strategies and in about 20 minutes the group realised that there is no real 'strategy' here. One can predict the winner simply by knowing how many individual chocolate squares are there. Then there were some more excited discussions about generalising the problem, and students ended up solving the generalised version: *if the chocolate bar has $n \times k$ blocks, then the total number of cuts it requires is $nk - 1$. So who wins is determined by whether nk is even or odd.*

After this we moved to problem 5. Here again, the students were excited about solving the problem and a couple of students figured out a solution in 5 minutes. We asked them to try and generalise the statement -

- what happens if the list if from 2 to 20? what happens if it is from 1 to 19?
- Is it really necessary to take numbers only from 1 to 20?
- Is it necessary to take a list of consecutive numbers?

By exploring such questions, the students came up with a general version of this game - *Make a list of any numbers that you like and play the game. The winner is determined by the number (specifically, parity) of odd numbers in the list.*

Problems 6 and 7 were quickly solved by most kids. We discussed how the solution can be expressed using the language of modular arithmetic.

The rest of the problems will be taken up during a subsequent session.

2 List of problems

1. Prove that the internal (or external) bisector of an angle of a triangle divides the opposite side internally (or externally) in the ratio of the sides containing the angle.

2. (Converse of Pythagoras' theorem) Prove that if in a ΔABC, $AC^2 = AB^2 + BC^2$ then ABC is a right-angled triangle with the right angle at B.

3. Find a point inside a convex quadrilateral such that the sum of the distances from the point to the vertices is minimal.

4. Two children take turns breaking up a rectangular chocolate bar 6 squares wide and 8 squares long. They may break the bar only along divisions between the squares. If the bar breaks into several pieces, they keep breaking the pieces up until only individual squares remain. The player who cannot make a break loses the game. Who will win?

5. The numbers 1 to 20 are written in a row. Two players take turns putting plus signs and minus signs between the numbers. When all such signs have been placed, the resulting expression is evaluated. The first player wins if the sum is even and the second one wins if the sum is odd. Who will win and how?

6. Prove that a power of 2 cannot end with 4 equal digits.

7. The sum of digits was calculated for 2^{100}, then the sum of the digits was calculated for the resulting number and so on, until a single digit is left. What digit is this?

8. Is it possible to write a perfect square using only the digits $2, 3, 6$ exactly 10 times each? How about if we use the digits $1, 2, 3$?