RAM Math Circle - Chennai

Synopsis for November 23 2025

Euler's function

In today's session we explored Euler's totient function. A number of students were familiar with the language of divisibility from earlier sessions. This session served as a reminder for some of the earlier results as well as opportunity to learn a new concept. We used the following problems to build the concept through exploration.

1. (Warm up) Prove that the product of any five consecutive natural numbers is divisible by 30, and by 120. (Students generalised this idea and realised that the product of n consecutive natural numbers would be divisible by $n!$).

2. Given a prime numbers p and q, find the number of natural numbers which are
   - Less than p and relatively prime to p.
   - Less than pq and relatively prime to it.
   - Less then p^2 and relatively prime to it.
   - Less than and relatively prime to p^k for any natural number k.
   - How about the same question for any natural number n? (Make a table of n and corresponding $\phi(n)$ for $n = 2, 3, \dots, 10$.

   Notation: For any positive integer n, the notation $\phi(n)$ denotes the number of positive integers less than and coprime to n.

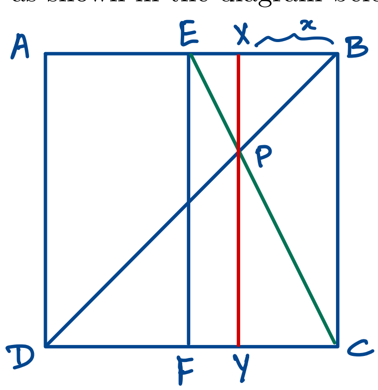
3. Suppose $(a, d) = 1$. Then for the set $S = \{a, a + d, a + 2d, \dots, a + (n-1)d\}$, number of numbers in S coprime to n is $\phi(n)$.

4. Suppose $(m, n) = 1$. What can you say about $\phi(mn)$? (To be explored during the next session).

Math with paper folding

We learned to fold the fraction 1/3 on the edge of a square piece of paper.

- Take a square piece of paper, and fold a vertical half line EF on it. Also fold one diagonal BD as shown in the diagram below.



Suppose length $XB = x$.

① $\Delta XPB \sim \Delta ADB \Rightarrow \frac{XP}{AD} = \frac{XB}{AB} \Rightarrow XP = x$.

② $\Delta BPE \sim \Delta DPC \Rightarrow \frac{EB}{PX} = \frac{DC}{PY}$

$\Rightarrow \frac{1/2}{x} = \frac{1}{1-x}$

$\Rightarrow \frac{1}{2}(1-x) = x \Rightarrow \boxed{x = 1/3}$

- Now construct the fold along segment EC. This one is a little tricky. For best results, pin down the paper at point E on one side with a finger, then pick up the corner B and manipulate the corner towards the segment EF and beyond while keeping the paper taut, till you identify a fold that joins E and C.

- Let P denote the intersection of BD and EC.
- Fold the segment XY that passes through P and is parallel to side BC.
- If we assume that the square has side of unit length 1, then length XB is equal to $1/3$.
- We proved this result using the similarity of two pairs of triangles as shown.

Now that you can fold $1/3$, you can think about how one may fold $1/5$, $1/7$ etc.