Problem 1. Ramesh the Tailor's Diagonal Detective Test. Ramesh the Tailor is proud of his new cloth-measuring trick! He folds a cloth along one diagonal, then the other, and if everything lines up he shouts, *"Perfect square!"* But his friend Shalu giggles, "Are you sure, Ramesh?" Can Ramesh be fooled? Is this folding test enough to guarantee the cloth is truly square?

Problem 2. Lakshmi's Number Train Mystery. Lakshmi drives a magical number train where each car carries the next natural number. One day she chooses exactly four consecutive train cars, adds the numbers inside them, and wonders aloud:

"Will the total ever be divisible by 4?"

Try out different sets of four cars and see whether Lakshmi ever gets lucky.

Then Lakshmi becomes even more curious:

"What if I take five consecutive cars instead? Will their total ever be divisible by 5?"

Experiment with several groups of five consecutive numbers and look for patterns.

Finally, Lakshmi challenges you:

"If I take k consecutive natural numbers, for which values of k does the total get divided evenly by k?"

Make a conjecture based on your explorations!

Problem 3. Aarav and Meera's Pebble Showdown. At the village fair, Aarav and Meera sit at a little table with two piles of 9 shiny pebbles. On each turn, a player may remove *any number* of pebbles from *exactly one* pile. Whoever takes the very last pebble jumps up and shouts, "I win!"

(a) If there are two piles of 9 pebbles, who can guarantee a win?

(b) What if there are three piles of 9 pebbles?

(c) What if there are four piles of 9 pebbles?

Problem 4. Priya's Magical Paper-Ripping Trick. Priya has a special sheet of paper that behaves strangely when she tears it. Each rip magically splits the sheet into either *4 pieces* or *6 pieces*. She boasts, "I can make exactly 9 pieces, or any bigger number too!" Is Priya telling the truth? Which numbers can she make using only these kinds of rips?

Problem 5. Prove that the numbers $16, 1156, 111556, 11115556, \ldots$ are all perfect squares.

Problem 6. The Square That Wanted Many Friends. A large square wants to turn itself into n little square friends. For $n \geq 6$, can it always cut itself into exactly n smaller squares (not necessarily equal)? Help the big square figure out which numbers of friends are possible.

Problem 7. The Bridge Puzzle of Towns A and B. Two towns, A and B, are on opposite sides of a canal with parallel straight sides. A road with a bridge that crosses the canal perpendicularly is to be built. Where should the bridge be located so that the length of the road from A to B be minimized?
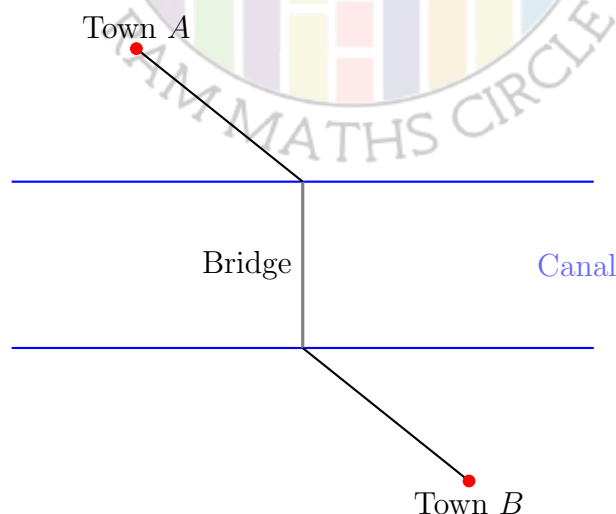
Problem 8. Which one is larger, $1 + 2 + 4 + 8 + 16 + 32 + 64 + \cdots + 2048$ or 4096?

Problem 9. Is it possible to write more than 50 different two-digit numbers on a blackboard without having two numbers on the board whose sum is 100?

Problem 10. Is it possible to find a number of the form $11\ldots1100\ldots00$ that is divisible by 2003? Is it possible to find a number of the form $11\ldots1$ that is divisible by 2003?

Problem 11. A straight bar of length 2m is cut into five pieces with each piece at least 17cm long. Prove that there are three of these pieces that can be put together to form a triangle.