Synopsis for October 12 2025

Triangles and similarity

The session was planned to be a review of similarity and an opportunity to work on slightly challenging problems on triangles and similarity.

Some students were very familiar with these topics but some weren't, so the students were split into two groups. The senior students worked on problems while some basic geometry concepts such as parallel lines and their properties, similarity of triangles etc. was revised with the junior students.

After the break, there was a discussion on the meaning of 'locus'. We discussed the examples of circle and parabola as a locus of points satisfying a certain geometric property. Kids had a lot of fun with the final folding activity, in which we folded a parabola using a square sheet of butter paper. The students have been asked to think about how the definition of a parabola as *the locus of points lying at an equal distance from a fixed point (focus) and a fixed line (directrix)* is reflected in the folding technique.

Here is a list of the results and problems covered.

Result 1 Any two parallelograms on the same base lying between the same pair of parallel lines have the same area. (The proof was provided in a fill-in-the-blanks form on the chalkboard.)
Remark: The area of all the above parallelograms equals that of a particular rectangle. This gives a formula for calculating the area of any parallelogram as

$$\text{(length of base)} \times \text{(perpendicular distance between the parallel lines)}.$$

Result 2 Suppose a triangle and a parallelogram lie on the same base and between the same pair of parallel lines. Then the area of the triangle is half that of the parallelogram.

The following were provided in a worksheet, which is to be solved in the next session.

Results

Result 1 If a straight line is drawn parallel to one side of a triangle, then it divides the other two sides proportionally. Conversely, If a straight line divides two sides of a triangle proportionally, then it is parallel to the third side.

Result 2 The internal (or external) bosector of a triangle divides the opposite side internally (or externally) in the ratio of the sides containing the angle.

Result 3 The areas of two similar triangles are proportional to the squares on the corresponding sides.

Resutl 4: Pythagoras' theorem: If ΔABC is a right angled triangle with the right angle at B, then $AC^2 = AB^2 + BC^2$.

Result 5 Converse of Pythagoras' theorem: If in a ΔABC, $AC^2 = AB^2 + BC^2$, then ΔABC is a right angles triangle with the right angle at B.

Let's solve

(Reference: "Challenges and thrill of pre-college mathematics")

1. $ABCD$ is a parallelogram and E is the midpoint of CD. Prove that the area of $\triangle ADE = \frac{1}{4}$ the area of parallelogram $ABCD$.
2. Consider the family $\mathcal{R}$ of parallelograms on equal bases whose areas are all equal. Prove that, in $\mathcal{R}$, that which is a rectangle has the least perimeter.
3. On the line segment AB, parallelograms $ABCD$ and $ABXY$ are drawn on opposite sides and AB or AB produced bisects CX. Then prove that $ABCD$ and $ABXY$ have equal areas.
4. ABC is a fixed triangle. P is any point on the same side of BC as that of A such that $\triangle PBC$ and $\triangle AABC$ have equal areas. Find the locus of P.
5. If the lengths of two sides of a triangle are given, show that its area is greatest when the angle between the sides is a right angle.

Homework

1. $ABCD$ is a given parallelogram. What is the locus of the remaining vertices of a parallelogram equal in area to $ABCD$, having AC as a diagonal?
2. Let $ABCD$ be a quadrilateral and X be the midpoint of BD. Prove that the area of $\triangle AXCB$ is one half the area of $ABCD$.
3. Let $ABCD$ be a quadrilateral. Suppose parallels are drawn through A and C to BD, and through B and D to AC. Prove that the resulting parallelogram has twice the area of $ABCD$.
4. If D is the midpoint of the side BC of $\triangle ABC$ and X is any point on AD, prove that the triangles AXB and AXC are of equal area.
5. ABC is a triangle and X is any point such that the area of $\triangle AXB =$ area of $\triangle XAC$. Find the locus of X.
6. $ABCD$ is a parallelogram and X is any point in the diagonal AC. Show that $\triangle ABX$ and $\triangle ADX$ are of equal area.
7. If two triangles have two sides of one equal to two sides of the other and the included angles are supplementary, prove that they are equal in area.
8. In quadrilateral $ABCD$, AC bisects BD. Show that AC also bisects the quadrilateral $ABCD$.