RAM Maths Circle

February 2, 2025

Nagpur

Session 1 : Overview

In this session, we taught combinatorics. The students approached the challenge creatively, devising solutions using the concept of permutation and combination in mathematics.

Problems and Solutions

Problem 1

Given the numbers $1, 2, 3, 4$**, how many 3-digit numbers can be formed without repetition? Similarly, using the letters** A, B, C, D, E**, how many 3-letter words can be formed without repetition?**

Solution: For the numbers: The first digit has 4 choices, the second has 3 choices, and the third has 2 choices.

$$4 \times 3 \times 2 = 24$$

For the letters: The first letter has 5 choices, the second has 4, and the third has 3.

$$5 \times 4 \times 3 = 60$$

Problem 2

In vehicle registration numbers, how many ways can the last four digits be formed if no repetition is allowed?

Solution: The first digit has 10 choices, the second has 9, the third has 8, and the fourth has 7.

$$10 \times 9 \times 8 \times 7 = 5040$$

Problem 3

There are 4 roads from school to the station, 7 roads from the station to home, and 15 direct roads from school to home. What is the total number of ways to reach home from school?

Solution: Paths through the station: $4 \times 7 = 28$. Direct paths: 15. Total ways:

$$28 + 15 = 43$$

Answer: 43 ways.

Problem 4

There are 4 ways to reach B from A, 6 ways to reach C from A, 2 ways to reach D from A, 5 ways to reach D from B, 2 ways to reach D from C, and 3 ways to reach C from B. What is the total number of ways to reach D from A?

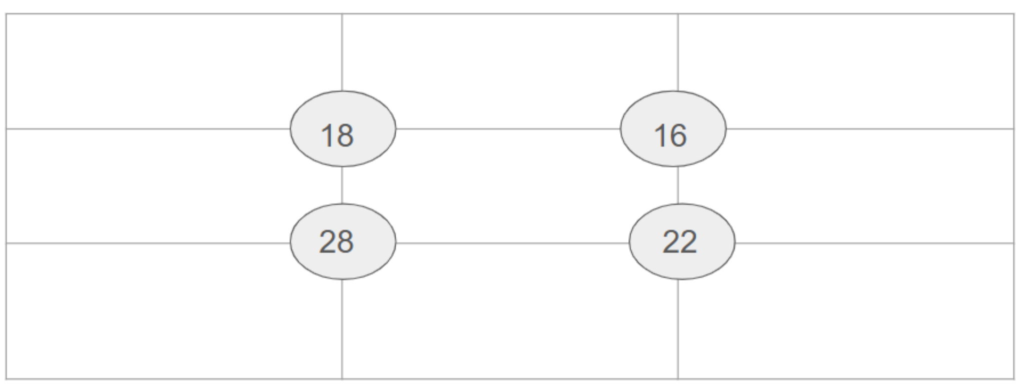
Solution: Ways to reach D from A directly: 2. Ways via B: $4 \times 5 = 20$. Ways via C: $6 \times 2 = 12$. Ways via B and then C: $4 \times 3 \times 2 = 24$. Total ways:

$$2 + 20 + 12 + 24 = 58$$

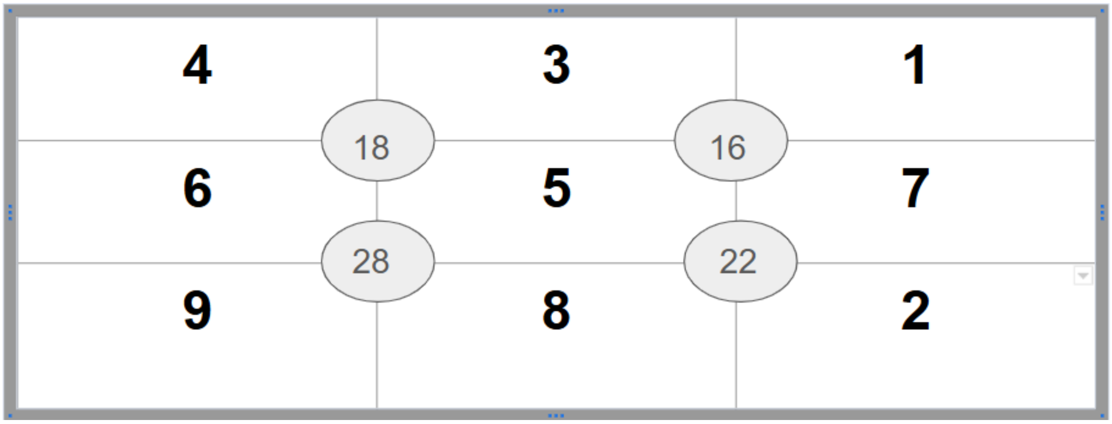
Answer: 58 ways.

Problem 5

Fill the numbers 1 to 9 in the grid with the given sums in the circles, ensuring no repetition.



Solution: One possible solution is:



Students explored more such possible solutions.

Practice Questions

Question 1

In how many ways can a committee consisting of 4 men & 2 women be chosen from 6 men & 5 women?

Solution Idea: Use the combination formula ${}^{n}_{r}$ separately for men and women and multiply the results.

Question 2

How many 3-letter words can be formed using FABLE?

Solution Idea: Use the permutation formula, as the order of letters mat- ters.

Question 3

In how many ways can 5 different books be arranged on a bookshelf?

Solution Idea: Use the factorial formula $n!$ to count the number of ways to arrange distinct items.

Question 4

A pizza restaurant offers 4 different toppings for their pizzas. If a customer wants to order a pizza with exactly 2 toppings, how many ways can this be done?

Solution Idea: Use the combination formula ${}^{n}_{r}$ where $n = 4$ and $r = 2$.

Question 5

A train is going from Mumbai to Pune and makes 5 stops on the way. Three persons enter the train after it has started

from Mumbai with 5 different tickets. How many different sets of tickets may they have had?

Solution Idea: Consider how each person can choose from the available tickets independently, leading to a counting problem.

Session 2 : Overview

During this session, we explored the Brick Wall Problem and its relation to the Fibonacci sequence and combinatorial methods.

0.1 Brick Wall Problem: Fibonacci Approach

We began by discussing how a wall of width n can be covered using bricks of size 1 and 2. By observing the patterns, we established the recurrence relation:

$$F(n) = F(n-1) + F(n-2), \tag{1}$$

with base cases $F(1) = 1$ and $F(2) = 2$. This directly follows the Fibonacci sequence.

0.2 Combinatorial Approach to the Brick Wall Problem

Next, we explored an alternative combinatorial approach. We analyzed how different combinations of 2-sized bricks and 1-sized bricks contribute to the total arrangements. The total number of ways is computed as:

$$\sum_{r=0}^{\lfloor n/2 \rfloor} \binom{n-r}{r}. \tag{2}$$

For $n = 10$, we calculated:

$$C(10, 0) + C(9, 1) + C(8, 2) + C(7, 3) + C(6, 4) + C(5, 5) = 89. \tag{3}$$

0.3 Staircase Problem and Fibonacci Sequence

We introduced another example where Fibonacci numbers naturally appear: the staircase problem. A person can take either 1 or 2 steps at a time to reach the *n*-th step. This results in the same Fibonacci recurrence relation:

$$F(n) = F(n-1) + F(n-2). \tag{4}$$

0.4 Lucas Numbers and Their Relationship with Fibonacci

Moving forward, we introduced the Lucas sequence:

$$L(0) = 2, L(1) = 1, L(n) = L(n-1) + L(n-2). \tag{5}$$

We then established a key relation between Fibonacci Series and Lucas Series:

$$L(n) = F(n-1) + F(n+1), \qquad \forall n. \tag{6}$$

Additionally, we explored another interesting pattern:

$$5F(n) = L(n-1) + L(n+1), \qquad \forall n. \tag{7}$$

We verified this pattern using specific values:

$$\begin{aligned} L(1) + L(3) &= 1 + 4 = 5 = 5F(2), \\ L(2) + L(4) &= 3 + 7 = 10 = 5F(3), \\ L(3) + L(5) &= 4 + 11 = 15 = 5F(4). \end{aligned}$$

Further Exploration

In upcoming sessions, we planned to explore additional Fibonacci and Lucas number relationships, including:

$$F(n+k) - F(n-k), \; L(n+k) - L(n-k). \tag{8}$$

References

0.3.1 https://polypad.amplify.com/lesson/brick-walls-and-fibonacci

https://r-knott.surrey.ac.uk/fibonacci/lucasNbs.html

(Overleaf Note:) The session was interactive, allowing students to find different approaches to solving problems. The exploration of mathematical in- duction and algorithmic construction deepened their understanding of combi- natorial techniques.