RAM Maths Circle
November 9, 2025
Nagpur

Introduction

Today's session was a lecture on recurrent relations. We began with the familiar example of arithmetic progression and used it to motivate the general idea of recurrence relations: a rule that defines each term of a sequence in terms of earlier term(s). The aim was to build intuition for forming base cases, to practise deriving recursive rules from patterns, and to explore several combinatorial and geometric examples where recurrence reasoning is natural.

Problem 1: Arithmetic Progression

We began with a familiar pattern to introduce the concept of recurrence relations.

Problem: Consider this sequence: 5, 8, 11, 14, ...

What is the next number in the sequence?

After establishing this example, students were challenged to create their own recurrence relation sequences. One example generated was: 1, 4, 11, 24, 45, ... where the relation is:

$$a_n = a_{n-1} + \left(n^2 - (n-1)\right) \quad \text{for } n \geq 2 \quad \text{with } a_1 = 1.$$

Key Components of a Recurrence Relation

From the examples above, we identified two essential components:

1. **Base case(s):** The initial value(s) needed to start the sequence (e.g., $a_1 = 5$)
2. **Recursive rule:** A formula that expresses each term in terms of previous terms (e.g., $a_n = a_{n-1} + 3$)

Problem 2: Bunny Hop

Problem. A bunny can hop either one or two steps. For a staircase of n steps ($n \geq 1$), how many distinct ways are there to reach step n? Make a table for small values $n = 1, 2, 3, 4, \ldots$ and use that table to conjecture a recurrence.

As an extension, students explored the variant where the bunny can hop 1, 2, or 3 steps, and were asked to state appropriate base cases.

Problem 3: Maximum regions in the plane

Consider an infinite plane. If we draw n lines dividing the plane into regions, what is the maximum number of regions possible?

Question: Make a table for $n = 1, 2, 3, 4, 5, 6, 7, \ldots$

Find a recursive relation for this problem.

Problem 4: The Circle Game (Josephus Problem)

During a historic event, Josephus and his 40 friends were playing a strategy game while stuck in a cave. They decided to play a counting game where they would form a circle and start eliminating every third remaining person from the game until only two players were left standing as winners.

General Problem: There are n people standing in a circle playing an elimination game. The counting begins at some point in the circle and proceeds around the circle in a fixed direction. In each step, $k - 1$ persons are skipped and the k-th person is eliminated from the game. The elimination proceeds around the circle (which is becoming smaller and smaller as the eliminated players step out), until only $k - 1$ persons remain, who are declared the winners.

We discussed the case where $k = 2$, meaning we eliminate every alternate person.

Let $J(n)$ denote the position of the survivor when there are n people initially.

Question: Calculate $J(n)$ for $n = 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 16$ and make a table.

Can we build a recurrence relation for this?

- Formulate a recurrence for the survivor index $J(n)$ in the $k = 2$ case.
- Use the table and parity observations to conjecture key properties.

Exploration

- Students practiced identifying patterns in sequences and translating them into recurrence relations. The concept that every recurrence relation requires a base case and a recursive rule was emphasized throughout.
- The arithmetic progression example introduced the basic structure of recurrence relations in a familiar context. Students then created their own sequences with different recursive rules, gaining hands-on experience in constructing recursive definitions.
- The Bunny Hop problem illustrated the power of bottom-up thinking. Students explored how to reach step n by considering possible previous steps, demonstrating how complex counting problems can be broken down by examining relationships between consecutive terms.
- The plane division problem connected geometric intuition with recurrence relations. Students investigated how adding each new line affects the total number of regions and worked to formalize their observations.
- The Josephus problem showcased how recurrence relations can model complex elimination processes. Students observed patterns in survivor positions, particularly noting properties related to even and odd values of n and powers of 2.

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