
IIIT Delhi - RAM Maths Circle

Session 10 and 11

(Organized by the Department of Mathematics, IIIT Delhi)

IIIT Delhi

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Simple Continued Fractions

- A simple continued fraction is an expression of the form

$$[a_0; a_1, a_2, \dots, a_N] = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{\ddots + \frac{1}{a_N}}}}$$

where $a_0 \in \mathbb{Z}$ and $a_1, a_2, \dots, a_N$ are positive integers. These $a_0, a_1, \dots, a_N$ are called *partial quotients*.

- If the number of terms is finite, it is called a *finite simple continued fraction*. If the expression continues indefinitely, it is called an *infinite simple continued fraction*.
- The n th convergent to the continued fraction $[a_0; a_1, a_2, \dots, a_N]$ is defined as

$$x_n = [a_0; a_1, a_2, \dots, a_n]$$

for all $0 \leq n \leq N$.

Problems

Problem 1. Expand the following rational numbers into finite simple continued fractions:

$$(i) \frac{17}{3}, \quad (ii) \frac{12}{17}, \quad (iii) -\frac{1}{5}.$$

Problem 2. Convert the following continued fractions into rational numbers:

- (i) $[2; 1, 4]$,
- (ii) $[-3; 2, 12]$,
- (iii) $[0; 1, 1, 100]$.

Problem 3. Can two different continued fractions represent the same number? If yes, give an example.

Problem 4. Prove that every finite simple continued fraction represents a rational number.
Hint: Use the principle of mathematical induction.

Problem 5. If x is representable by a simple finite continued fraction with an odd (even) number of convergents, it is also representable by one with an even (odd) number of convergents.

Problem 6. Evaluate the infinite simple continued fractions

$$[1; 1, 1, 1, \dots] \quad \text{and} \quad [2; 4, 4, 4, \dots].$$

Problem 7. Expand each of the following as infinite simple continued fractions:

$$(i) \sqrt{2}, \quad (ii) \sqrt{2} - 1, \quad (iii) \sqrt{3}.$$

Problem 8. If

$$\pi = [3; 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, \dots],$$

compute the first 5 convergents.

Problem 9. Let $[a_0; a_1, a_2, \dots, a_N]$ be a simple continued fraction. If we define

$$p_0 = a_0, \quad p_1 = a_0 a_1 + 1,$$

$$p_n = a_n p_{n-1} + p_{n-2} \quad \text{for } 2 \leq n \leq N,$$

and

$$q_0 = 1, \quad q_1 = a_1,$$

$$q_n = a_n q_{n-1} + q_{n-2} \quad \text{for } 2 \leq n \leq N,$$

then prove that

$$x_n := [a_0; a_1, \dots, a_n] = \frac{p_n}{q_n}$$

for all $0 \leq n \leq N$.

Problem 10. If p_n and q_n are as defined in Problem 9, prove that

(i)

$$p_n q_{n-1} - q_n p_{n-1} = (-1)^{n-1} \quad \text{for all } n \geq 1.$$

(ii)

$$p_n q_{n-2} - q_n p_{n-2} = (-1)^n a_n \quad \text{for all } n \geq 2.$$

(iii)

$$q_n \geq q_{n-1} \quad \text{for all } n \geq 1.$$

(iv)

$$q_n \geq n \quad \text{for all } n \geq 2.$$