RAM Maths Circle

December 7, 2025

Nagpur

Introduction

The session commenced with an engaging warm-up activity focused on the Collatz conjecture and its intriguing connection to the Fibonacci sequence, encouraging students to observe patterns. Following this, the discussion shifted to the fundamental properties of triangles, covering essential concepts such as the angle sum property, the exterior angle property, and the triangle inequality theorem. Each property was carefully explained, after which students practiced applying these concepts through a series of problem-solving exercises.

Problems

Problem 1: Collatz-Fibonacci

Consider the following modified Collatz process:

- If a number n is odd, replace it with $n+1$.
- If a number n is even, replace it with $\frac{n}{2}$.

It is easy to verify that every positive integer eventually reaches 1 under this process, after which the sequence enters the repeating cycle

$$2 \to 1 \to 2 \to 1 \to \cdots$$

An interesting pattern emerges when we count how many starting numbers reach 1 in exactly n iterations.

Claim. After n iterations of this modified Collatz process, exactly

$$F_{n+1}$$

positive integers reach the value 1, where F_{n+1} denotes the $(n+1)$-th Fibonacci number.

Problem 2: Quadrilateral Inequality

Prove that in any quadrilateral $ABCD$, the sum of its four sides is greater than the sum of its two diagonals; that is,

$$AB + BC + CD + DA > AC + BD.$$

Problem 3:

Triangles ABC and ADC are isosceles with $AB = BC$ and $AD = DC$. Point D lies inside $\triangle ABC$. You are given that

$$\angle ABC = 40^\circ \quad \text{and} \quad \angle ADC = 140^\circ.$$

Find the measure of $\angle BAD$.

Problem 4:

In triangle ABC, suppose that $\angle BAC = 90^\circ$. Let AD be the altitude from A to side BC. Prove that

$$\angle BAD = \angle ACB.$$