IIIT Delhi - RAM Maths Circle
Session 4

(Organized by the Department of Mathematics, IIIT Delhi)

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§1. Counting Principle

Combinatorics, a branch of mathematics that is devoted to the art of counting things, is an enjoyable topic that has many applications in mathematics and computer science. At the same time, working on a good combinatorics problem is as close to puzzle solving as math can get.

For the next several lessons, we will be studying combinatorics. While we will learn several useful formulas, our primary goal is not to memorize them, but to develop an understanding of where these formulas come from. We also will be solving problems that promote understanding of combinatorics principles, not the direct application of the formulas.

Some warm-up problems:

1. (**Mithu the Talking Parrot**) Mithu is a talking parrot who always speaks in three-word sentences. Each of Mithu's sentences starts with a pronoun, followed by a verb, and then a noun. Mithu knows:

   - 2 pronouns: **I** and **WE**,
   - 3 verbs: **LOVE**, **WANT**, and **COOK**,
   - 4 nouns: **FOOD**, **CRACKER**, **FRIEND**, and **BOOK**.

   How many different sentences can Mithu the Parrot say?

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2. (**Albela the Chatty Parrot**) Mithu's friend, Albela the Parrot, can talk as well. Each of Albela's sentences starts with an adjective, followed by a noun, and then a verb. Albela knows:

   - 3 adjectives: **HAPPY**, **HUNGRY**, and **LONELY**,
   - 2 nouns: **PARROT** and **CROCODILE**,
   - 3 verbs: **SINGS**, **CRIES**, and **WORKS**.

(a) How many different sentences can Albela the Parrot say?

(b) Mithu and Albela decide to create a two-sentence story, where each parrot contributes one sentence. How many different stories can they come up with?

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3. (**The Stylish Lady Zara**) Lady Zara has a keen sense of fashion. She owns:

   - 3 different black skirts,
   - 5 different jackets — 3 blue and 2 green,
   - 10 different hats — 6 blue and 4 green.

   An *outfit* for Lady Zara consists of one skirt, one jacket, and one hat, where the jacket and hat must be of the **same color**. In how many different ways can Lady Zara choose her outfit?

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4. A number is called *super-odd* if all its digits are odd (for example, 5, 33, 13573). How many three-digit super-odd numbers with all digits different are there?

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5. Suppose that we want to have 10 girls sitting in 10 chairs placed in a row. In how many ways can we do it?

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6. Call a positive integer *nice* if it contains only even digits.

   (a) Write down all the nice two-digit numbers; how many are there?

   (b) How many five-digit numbers are nice?

   (c) How many six-digit numbers have at least one even digit?

   (d) Which are there more of: seven-digit numbers that contain a 1 or seven-digit numbers that have no 1's.

Some thinking problems:

1. In how many ways can you rearrange the numbers from 1 to n so that

   (a) neither 1 nor 2 occurred in its original position;

   (b) exactly one of the numbers $1, 2$, and 3 stayed in its original position;

   (c) none of the numbers $1, 2$, and 3 occurred in their original positions;

(d) none of the numbers $1, 2, 3$, and 4 occurred in its original position?

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2. Fifteen students sit on fifteen numbered chairs. Every minute a kind teacher moves them according to the following scheme:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 3 & 5 & 10 & 8 & 11 & 14 & 15 & 6 & 13 & 1 & 4 & 9 & 7 & 2 & 12 \end{pmatrix}$$

In how many minutes will all the students be in their original places again?

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3. (**This is not a counting problem**) Each letter in HAGRID's name represents a distinct digit between 0 and 9. Show that

$$HAGRID \times H \times A \times G \times R \times I \times D$$

is divisible by 3. For example, if $H = 1, A = 2, G = 3, R = 4, I = 5, D = 6$, then $HAGRID \times H \times A \times G \times R \times I \times D = 123456 \times 1 \times 2 \times 3 \times 4 \times 5 \times 6$.

Students should keep in mind that even if a problem seems difficult at first, it is always worthwhile to return to it later with fresh ideas. Avoid the temptation to flip straight to the solutions! And remember, some problems may also have alternative solutions that do not rely on usual counting or permutation principle.