Synopsis for February 08 2026

Junior Batch

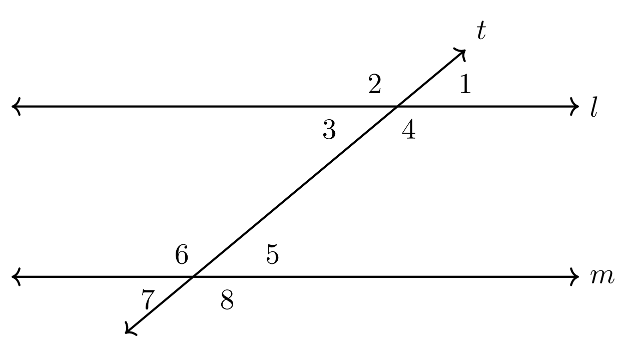
We began this session by reviewing key ideas from the previous class (January 24th) and solving problems involving angles. We emphasized a systematic approach to geometry:

- **Visualization:** Always begin by drawing a clear and reasonably accurate diagram.
- **Algebraic Translation:** Convert geometric relationships into algebraic equations. For example, assign an unknown angle the variable x and form equations accordingly.
- **Auxiliary Constructions:** Many problems require adding extra lines to the diagram. Common constructions include:
  - Drawing a line parallel to a given line through a specific point,
  - Extending a ray to form a complete line,
  - Dropping a perpendicular,
  - Drawing an angle bisector.
- *Remark:* Choosing the correct construction is often the most challenging part. We focused on identifying clues in the problem statement that suggest a suitable construction.

Parallel Lines and Transversals

We first recalled the proof that **vertically opposite angles are equal**, and then moved to properties of parallel lines.

- **Definition:** A **transversal** is a line that intersects two or more coplanar lines at distinct points.
- **Axiom (Corresponding Angles):** If a transversal intersects two parallel lines, then corresponding angles are equal.
- **Theorem (Alternate Interior Angles):** If a transversal intersects two parallel lines, then alternate interior angles are equal.



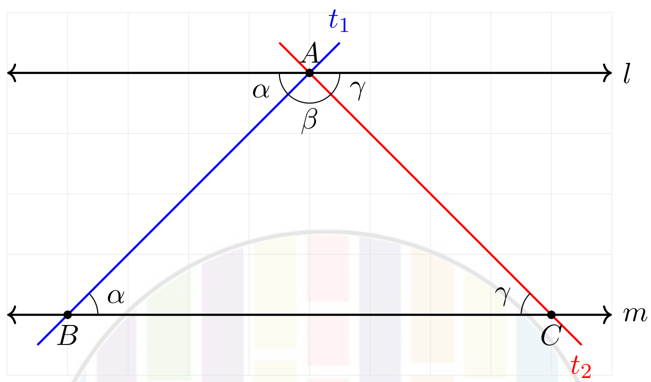
In the figure above, $l \parallel m$.

Angle Relationships:

- **Corresponding Angles:** $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, $\angle 4 = \angle 8$.
- **Vertically Opposite Angles:** $\angle 1 = \angle 3$, $\angle 5 = \angle 7$, etc.
- **Alternate Interior Angles:** Since $\angle 3 = \angle 1$ (vertical angles) and $\angle 1 = \angle 5$ (corresponding angles), it follows that $\angle 3 = \angle 5$.

Triangle Sum and Exterior Angles

We explored triangle properties using two transversals intersecting two parallel lines.



Deductions:

1. **Angle Sum Property:** At vertex A, the straight angle on line l gives

$$\alpha + \beta + \gamma = 180°.$$

   Since α and γ are alternate interior angles equal to the base angles of the triangle, it follows that the sum of the interior angles of a triangle is 180°.

2. **Exterior Angle Theorem:** If a side of a triangle is extended, the exterior angle formed equals the sum of the two remote interior angles.

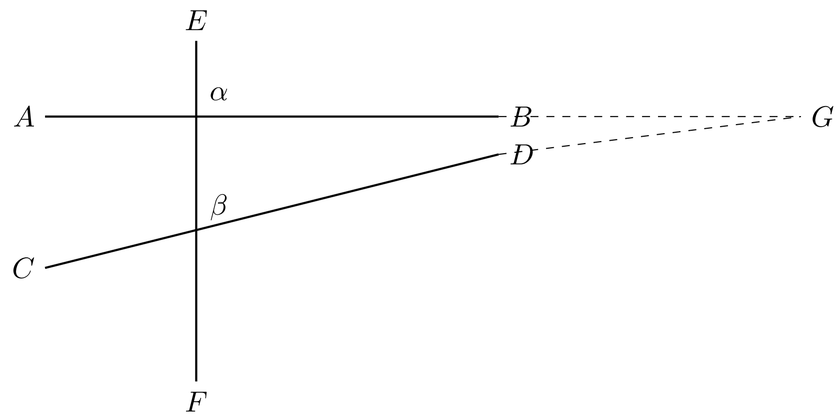
Converse: Proving Lines Are Parallel

We discussed the converse statement:

If two lines are cut by a transversal such that corresponding angles are equal, then the lines are parallel.

Proof (by Contradiction):
Assume lines AB and CD are *not* parallel, even though corresponding angles satisfy $\alpha = \beta$.

- If the lines are not parallel, they intersect at some point G.
- Together with the transversal EF, they form a triangle.



In the resulting triangle, the exterior angle α equals the sum of the two remote interior angles:

$$\alpha = \beta + \angle G.$$

But we are given $\alpha = \beta$, which implies

$$\angle G = 0^\circ,$$

a contradiction. Therefore, the lines must be parallel.

Senior Batch

We continued our discussion of polynomials by recalling the division algorithm. Then we proceeded to the remainder theorem, first verifying it through examples to understand the mechanics, and then proving it.

The remainder theorem: Let $p(x)$ be a polynomial with real coefficients, and α be a real number. The remainder obtained by dividing $p(x)$ by $(x-\alpha)$ is $p(\alpha)$.

This result seems very surprising when it is first stated, but after proving it using the division algorithm, it appears as a very natural outcome.

After this we proceeded to analyze why synthetic division works in the case when the divisor is a degree 1 polynomial.

Suppose we wish to divide the polynomial $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ by $(x-\alpha)$. Then by the division algorithm,

$$p(x) = (x-\alpha)(b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + \cdots + b_1 x + b_0) + r.$$

Notice that the degree of the divisor $(x-\alpha)$ is 1, so the quotient must have degree $(n-1)$ and the remainder must have degree 0. We expand out the right-hand-side term and get

$$b_{n-1}x^n + b_{n-2}x^{n-1} + \cdots + b_1x^2 + b_0x - \alpha b_{n-1}x^{n-1} - \alpha b_{n-2}x^{n-2} - \cdots - \alpha b_1 x - \alpha b_0 + r.$$

Now equate both sides:

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 = b_{n-1}x^n + (b_{n-2} - \alpha b_{n-1})x^{n-1} + \cdots + (b_1 - \alpha b_2)x^2 + (b_0 - \alpha b_1)x - \alpha b_0 + r.$$

Equating coefficients of each x^i we get the following set of equations. Since the polynomial $p(x)$ is known, the coefficients a_i are known; α is also known and we want to find the coefficients b_i of the quotient and also the remainder. So we write our equations accordingly:

$$
\begin{aligned}
b_{n-1} &= a_n \\
b_{n-2} - \alpha b_{n-1} &= a_{n-1} \\
&\vdots \\
b_1 - \alpha b_2 &= a_2 \\
b_0 - \alpha b_1 &= a_1 \\
r - \alpha b_0 &= a_0
\end{aligned}
$$

If we compute the values of b_i's and r from these equations we get:

$$
\begin{aligned}
b_{n-1} &= a_n \\
b_{n-2} &= a_{n-1} + \alpha b_{n-1} \\
&\vdots \\
b_1 &= a_2 + \alpha b_2 \\
b_0 &= a_1 + \alpha b_1 \\
r &= a_0 + \alpha b_0
\end{aligned}
$$

Using these equations we can outline a procedure to find the quotient and remainder: first $b_{n-1} = a_n$, use this to compute $b_{n-2} = a_{n-1} + \alpha b_{n-1}$, then use these two terms to compute $b_{n-3} = a_{n-2} + \alpha b_{n2}$ and so on till in the end we get $b_0 = a_1 + \alpha b_1$ and $r = a_0 + \alpha b_0$.

This procedure can be visualised by writing the coefficients a_i's in a row and the computed terms αb_i's right below the respective a_i's and adding them in turn to produce the respective b_{i-1}'s.

α	a_n	a_{n-1}	a_{n-2}	$\cdots$	a_2	a_1	a_0
		αb_{n-1}	αb_{n-2}		αb_2	αb_1	αb_0
	b_{n-1}	b_{n-2}	b_{n-3}		b_1	b_0	r

The students had fun with this part and it was a nice moment of understanding when they could see *why* the method they learnt in school actually works!

Now that we had understood this idea, we decided to explore the same with a divisor of degree 2, such as $x^2 + cx + d$. One of the students, Param, noted that we need not assume that the highest degree coefficient in the divisor is always 1, i.e. we can take our divisor to be of the form $cx + d$ or $cx^2 + dx + e$. This would only require an extra division by that coefficient in the end, which is indeed true!

Param also proceeded to try and generalise this exploration by trying to see if we can use the same idea to find a general procedure to divide a degree n polynomial by any degree m polynomial (where $m < n$). We agreed to continue this exploration next week!