
IIIT Delhi - RAM Maths Circle
Session 22 & 23

(Organized by the Department of Mathematics, IIIT Delhi)

IIIT-Delhi

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Problem 1: Compute:

$$1^2 + 0^2, \quad 1^2 + 1^2, \quad 2^2 + 1^2, \quad 2^2 + 2^2, \quad 3^2 + 2^2.$$

- (i) What numbers do you get?
- (ii) Which numbers up to 20 can be written as a sum of two squares?
- (iii) Which numbers cannot?

Problem 2. Try to express the following as a sum of two squares (if possible):

$$7, 11, 15, 21, 25, 29.$$

- (i) Which ones work?
- (ii) Look at these numbers modulo 4. Do you see a pattern?

Problem 3. Based on your observations, complete the statement:

Numbers of the form _____ can be written as a sum of two squares.

Test your conjecture on:

$$33, 37, 41, 43.$$

Can you prove your statement? Before proving that look at the square of a number modulo 4.

Problem 4. Expand:

$$(a^2 + b^2)(c^2 + d^2).$$

- (i) Can you rewrite the result as a sum of two squares?
- (ii) What does this tell you about products of numbers that are sums of two squares?

Problem 5. Suppose $m = a^2 + b^2$ and $n = c^2 + d^2$.

- (i) Use the previous problem to show that mn is also a sum of two squares.
- (ii) Try an example with actual numbers.

Problem 6. Check the primes:

2, 3, 5, 7, 11, 13, 17, 19.

- (i) Which primes can be written as a sum of two squares?
- (ii) Separate them into two types: $4k + 1$ and $4k + 3$.
- (iii) What do you notice?

Problem 7. Try to write the following numbers as sums of two squares:

50, 65, 85, 97.

- (i) Can you use factorization and previous results to help?

Problem 8. Try expressing the following as sums of three or four squares:

6, 7, 14, 15, 23.

- (i) Do you think every number is a sum of three squares?
- (ii) Do you think every number is a sum of four squares?
- (iii) Make a conjecture.

Problem 9. Write the following numbers as sums of consecutive positive integers:

9, 15, 21, 25, 27.

Problem 10. Try:

8, 16, 32.

- (i) Can these be written as sums of consecutive positive integers?
- (ii) What is special about these numbers?

Problem 11. Explore more numbers:

12, 18, 20, 45.

- (i) Find at least two different ways (if possible) to write each number.
- (ii) What do you notice about numbers that allow many representations?

Problem 12. Complete the statement:

Every number that has _____ can be written as a sum of consecutive positive integers.

Test your conjecture on:

