RAM Maths Circle
August 10, 2025
Nagpur

Introduction

In this engaging session, participants delved into two captivating mathematical challenges that blended number theory, optimization, and strategic gameplay. The first problem explored the partition product, encouraging students to uncover patterns and maximize outcomes through creative partitioning of numbers. The second introduced the Femto Card Game, a dynamic exercise in strategic thinking, where players honed their decision-making skills to outwit their opponents. These activities fostered critical thinking, collaboration, and a deeper appreciation for the elegance of mathematical structures and game theory.

Problem 1: Partition Product (PP)

Problem: The partition product $PP(n)$ is defined as the maximum product of positive integers whose sum is n. For example:

- For $n = 10$, one partition is $10 = 3 + 3 + 3 + 1$, with product $3 \cdot 3 \cdot 3 \cdot 1 = 27$.

- Another partition is $10 = 2 + 2 + 2 + 2 + 2$, with product $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$.

The goal is to find the partition that yields the maximum partition product for a given n, such as $PP(25)$ and $MPP(100)$, and explain why the value obtained is the maximum. Why are powers of 3 often considered in the solution?

Exploration

Students began by experimenting with small numbers, calculating partition products for values like $n = 5, 10$, and 15. They tested various partitions, such as splitting 10 into $5 + 5$, $4 + 3 + 3$, or $2 + 2 + 2 + 2 + 2$, and compared the resulting products. As they progressed to larger numbers like $n = 25$ and $n = 100$, they noticed recurring patterns, particularly the dominance of terms involving 3's and occasional adjustments with 2's or 4's. Through group discussions, they explored why partitions with numbers close to 3 consistently yielded higher products, identifying connections between the number of terms and their values.

Problem 2: Femto Card Game

Problem: Femto is a card game for two players with a deck of eight cards numbered 2, 3, 4, 5, 6, 7, 8, 10. The cards are shuffled and dealt, so each player gets four cards. In each round, both players play one card face down, which are then revealed. The higher card wins the round, unless it is more than twice the value of the lower card, in which case the lower card wins (e.g., 10 beats 8, 6 beats 5, 3 beats 10, 10 beats 5). The winner chooses one of the two cards as trophy while the loser takes the remaining card back into their hand. Rounds continue until one player has no cards left. The winner is the player with the higher total value of cards in front of them at the end. Assuming both players play optimally, devise a strategy to maximize the score and determine the winner.

Exploration

Students were divided into groups, with one member acting as a scribe to record moves, while two others played the Femto Card Game. They simulated multiple rounds, experimenting with different card choices to understand the impact of the game's unique rule where a lower card can win if the higher card exceeds twice its value. By analyzing outcomes, groups attempted to develop strategies to maximize their score, such as choosing when to play high cards like 10 or low cards like 2 to exploit the rule. They discussed how to balance winning rounds with securing high-value cards, refining their approaches through trial and error to identify optimal strategies.