
IIIT Delhi - RAM Maths Circle

Session 7

(Organized by the Department of Mathematics, IIIT Delhi)

IIIT Delhi

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Problem 1: Two players play the following game.

There are m stones in a pile. On each turn, a player may remove any number of stones from 1 through k . The player who removes the last stone wins.

- (a) For $k = 3$, list the winning and losing positions for $m = 1, 2, \dots, 21$.
- (b) Find a pattern in the losing positions.
- (c) For general k , conjecture which values of m are losing positions.
- (d) Prove your conjecture.
- (e) Suppose $m = 2026$ and $k = 7$. Does the first player have a winning strategy? If so, what is the first move?

Problem 2: There are two piles containing a and b stones.

On each turn, a player may remove any positive number of stones from exactly one pile. The player who removes the last stone wins.

- (a) Determine whether each of the following positions is winning or losing:

$(1, 1), (1, 2), (2, 2), (2, 3), (3, 3), (3, 4), (4, 4), (4, 5)$.

- (b) Find and explain a pattern in the positions.
- (c) Prove your answer for positions of the form (n, n) .
- (d) For a general position (a, b) , determine when the position is losing.
- (e) Describe a strategy that allows the winning player to maintain a losing position for their opponent after every move.

Problem 3 (a): Two criminals, Alice and Bob, are arrested for committing a crime together. The police do not have enough evidence to prove that they committed the crime, so they separate the two prisoners and question them independently.

Each prisoner has two choices:

- **Stay Silent** — refuse to confess.
- **Snitch** — confess and testify against the other prisoner.

The sentences are determined as follows:

- If both stay silent, they are each sentenced to 1 year in prison.
- If one snitches and the other stays silent, the snitch is released immediately while the silent prisoner receives 5 years.
- If both snitch, they are each sentenced to 3 years.

Answer the following questions accordingly.

- If Bob stays silent, what should Alice do?
- If Bob snitches, what should Alice do?
- What is Alice's best strategy? What about Bob's?
- Does either prisoner have a dominant strategy?
- Find the Nash equilibrium.
- Is the Nash equilibrium the outcome in which the prisoners receive the smallest total sentence? Explain.

Problem 3 (b): Alice and Bob are going to the cinema together. They have not decided which movie to watch. Each person secretly chooses between Movie A and Movie B. If they choose the same movie, they go together and both enjoy the evening. Alice prefers Movie A, while Bob prefers Movie B. If they choose different movies, they cannot agree and the evening is ruined, so both receive 0 points.

The payoff matrix is given below.

	Bob: Movie A	Bob: Movie B
Alice: Movie A	(3, 2)	(0, 0)
Alice: Movie B	(0, 0)	(2, 3)

- Find all Nash equilibria.
- Why do you think there is more than one Nash equilibrium?
- Does either player have a dominant strategy?

Challenge Problem: Consider the rivalry between Airbus and Boeing to develop a new commercial jet aircraft.

Suppose Boeing is ahead in the development process and Airbus is considering whether to enter the competition.

If Airbus stays out, it earns 0 profit, whereas Boeing enjoys a monopoly and earns a profit of \$1 billion.

If Airbus decides to enter and develop the rival airplane, then Boeing has to decide whether to accommodate Airbus peacefully or to wage a price war.

In the event of peaceful competition, each firm will make a profit of \$300 million.

If there is a price war, each firm will lose \$100 million because the prices of airplanes will fall so low that neither firm will be able to recoup its development costs.

1. Draw the game tree for this game.
2. Find the rollback equilibrium.
3. Describe the firms' equilibrium strategies.

Some Important Terms

Game Theory: The study of how people or organisations make decisions when the outcome depends on the decisions of others.

Player: A person, group, or organisation making decisions in a game.

Strategy: A complete plan describing what a player will do at every point where they may have to make a decision.

Payoff: The outcome or reward a player receives from a particular combination of choices. Payoffs can represent money, points, years in prison, or preferences.

Game Tree: A diagram showing the sequence of decisions, actions, and possible outcomes in a sequential game.

Best Response: The strategy that gives a player the highest payoff given the other player's choice.

Dominant Strategy: A strategy that gives a player a better payoff than any other strategy, regardless of what the other player does.

Nash Equilibrium: A set of strategies in which no player can improve their payoff by changing their strategy alone, assuming the other players keep their strategies unchanged.

Backward Induction: Solving a sequential game by starting at the final decisions and working backwards to determine what players will rationally choose.

Rollback Equilibrium: The outcome and strategies obtained by applying backward induction to a sequential game.